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Ambiguity in Securitization Markets

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Abstract

During the financial crisis of 2008, origination and trading in asset-backed securities markets dropped dramatically. I present a model with ambiguity averse investors to explain how such a market freeze could occur and to investigate how ambiguity affects origination and securitization decisions. The model captures many features of the crisis, including market freezes and fire sales, as well as the timing and duration of the freeze. The presence of ambiguity also reduces real economic activity. Lastly, I consider the differing implications of ambiguity and risk, as well as the role of policies that reduce ambiguity during market freezes.

Keywords: securitization; ambiguity aversion; market freezes

JEL Classification: G01, G21, G28, E44

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1. Introduction

Financial intermediation has traditionally involved banks taking deposits and making loans to firms and consumers. Recently, however, the intermediation process has become market-based, in that banks also borrow money, securitize loans, and buy and sell securities in the open market. This change was facilitated by the development of significant markets in asset-backed securities (ABS) and collateralized debt obligations (CDOs).[1] This securitization process has created securities that, prior to the financial crisis of 2008, were in high demand by investors.

However, during the recent financial crisis, many markets, including those of asset-backed securities and CDOs, suffered from a lack of origination. In October 2008, the volume in these markets essentially went to zero, as investors were unwilling to participate at any offered price. Additionally, issuance of new securities ceased in many asset classes of ABS, as shown in Figure 1. Contrary to standard economic theories, which suggest that the market should be able to find an equilibrium price, trade halted in these markets. ABS markets did not begin to recover until early 2009 when the Federal Reserve offered support programs to encourage investor participation. However other markets, such as those of CDOs, which were not supported by any government programs during the crisis, have still not recovered, as seen in Figure 2.

This paper develops a model of financial intermediation to explain the market freeze seen in securitization markets during the crisis using ambiguity aversion. The first goal of the model is to analyze how ambiguity aversion among investors influences both their participation decision and the banks' securitization decision. While previous work has shown how ambiguity can lead to limited participation by investors, this model looks more specifically at the context of securitization markets. Securitization markets may be more susceptible to ambiguity given the complexity of the structure of these securities. Asset-backed securities are comprised of many underlying assets with complicated default probabilities and correlations, which make assessing the value of these securities difficult for some investors. This can be true even with full information disclosure given time, modeling, and processing limitations. This is potentially why markets trading ABS and CDOs completely collapsed during the crisis, while other markets were more resilient. Further, the additional complexity of CDOs may contribute to their continued

[1] ABS markets trade securities whose values are based on the cash flows of underlying assets such as mortgages (MBS), car loans, student loans, or credit card receivables. CDO markets trade securities whose values are based on the cash flows of underlying assets such as high yield corporate loans (CLOs), high yield and investment grade corporate bonds (CBOs), and other securitized products such as ABS, MBS, or other CDOs.

stagnation. The second goal of the model is to understand how real investment, in terms of the number of loans undertaken, is influenced by the possibility of facing ambiguity averse investors.

The role of ambiguity aversion is illustrated by the Ellsberg paradox.[2] Ellsberg (1961) shows that when individuals face uncertainty, they do not act as if they have a single prior, thus violating the independence axiom of Expected Utility Theory discussed by von Neumann and Morgenstern (1947) and Savage (1954). The distinction between risk and ambiguity was first discussed by Knight (1921) and was formally developed by Schmeidler (1989) and Gilboa and Schmeidler (1989). They suggest that an individual facing uncertainty will use their minimum expected utility when making a decision. That is, from among his set of priors, the individual will consider the one that yields the worst-case outcome. This max-min utility will characterize ambiguity averse investors in my model.

The economy considered in this paper consists of banks, investors, and depositors. Banks can originate risky loans with normally distributed returns. They can then either retain these loans or securitize and sell them to risk averse and ambiguity averse investors. There are two states of the world – one in which investors face ambiguity and one in which they do not. Given the possibility of facing ambiguity averse investors, who may choose not to participate if ambiguity is severe enough, banks originate fewer loans than they would otherwise.

The model captures many significant features of the crisis that existing explanations of market freezes, including adverse selection, regulatory arbitrage, and neglected risks, also capture. However, ambiguity aversion also depicts an important additional aspect – that is, both the timing and, in particular, the duration of the crisis. Some existing theories suggest that, once bad news comes into the market, a freeze can occur. However, despite negative information on the housing market beginning to surface in mid-2007, securitization markets did not freeze until late 2008. Additionally, while existing theories can explain how a liquidity crisis can occur, they do not explain how one can persist. My model suggests that, given the presence of ambiguity, a

[2] In Ellsberg's experiment, an individual is presented with two urns. Urn one has 50 black balls and 50 red balls. Urn 2 has 100 balls in some unknown proportion of black and red. When given the choice between $100 if a black ball is drawn from urn 1 and $0 otherwise, and $100 if a black ball is drawn from urn 2 and $0 otherwise, most individuals choose the former gamble. This implies that the expected probability of black balls in urn 2 is less than 0.5. When given the same choices except with a payoff if a red ball is drawn, most individuals will again choose the gamble with urn 1. This implies that the expected probability of red balls in urn 2 is less than 0.5. However, the probability of both black and red cannot be less than 0.5, suggesting that the individual has multiple priors due to the uncertainty of the distribution.

market freeze can persist as long as investors remain uncertain about the expected outcome. This helps explain why, even several years after the initial crisis, CDO markets are still stagnate despite limited losses on these securities.

The model also captures another significant feature of the crisis, namely that it was not just the financial markets that froze. Origination of new loans also declined in the real economy. Banks did not invest in new loans, as they could not find buyers for their newly-created securities and had limited funding since they were stuck holding older loans that they could not sell. An important aspect of this model is the delay between when banks can originate loans and when they can sell them to investors. As discussed by Jiang et al. (2014), there is often a several-month gap between when a loan is originated and when it is sold given the time it takes to acquire and tranche a pool of securities. This delay is longer for non-conforming, higher risk, and other non-standard loans. The information revealed to investors during this time will influence the amount they are willing to purchase and the price they are willing to pay. In this model, the level of ambiguity is unknown during origination but is revealed prior to securitization. This can result in banks being left holding their loans if markets dry up, which is consistent with what happened during the crisis (Erel et al. (2014)). As a result, ambiguity can have significant real effects on the supply of loans ex post once ambiguity is realized.

However, even more notable is that the presence of ambiguity can also reduce origination ex ante. If banks anticipate that investors will face ambiguity and therefore be less willing to participate in the secondary market, banks will originate less in the primary market in response. An active secondary market can increase origination by providing an additional source of funding for originators. However, ambiguity can reduce this benefit. Increased reliance on securitization can lead to lower origination if investors are ambiguity averse, even if the state with high ambiguity is not realized.

In addition to its impact on origination, the model shows how the presence of ambiguity can affect the level of securitization. In many cases, banks securitize more of their loans when investors do not face ambiguity. Securitization in the state with ambiguity is significantly reduced when ambiguity is severe, additional investment opportunities are less profitable, or expected payoffs are low. However, there are also cases in which securitization is high when investors face ambiguity. In these cases, the potential ambiguity faced by investors is relatively mild or the payoffs to securitization are relatively high. Accordingly, banks originate a large number of loans funded by deposits. In order to repay their debt, banks may be forced to sell securities at fire sale prices if the state with ambiguity is realized. Therefore, the model can create both market freezes and fire sales, depending on the banks' debt constraint, which is in turn determined by the prevalence of ambiguity and the attractiveness of securitization.

The model also lends itself to a comparison of the differing impacts of risk and ambiguity. Both increased risk, as measured by the variance of the distribution of returns, and increased ambiguity will reduce the price investors are willing to pay for securities. However, the bank can counteract the negative affect of risk by selling fewer securities because price is a function of both risk and quantity. The price impact of higher ambiguity, on the other hand, is strictly negative. Therefore, while the impact of risk on securitization is gradual, the impact of ambiguity is more abrupt. Because of this, in certain regions, a small change in the level of ambiguity can have a large effect on the origination and securitization decisions of the bank. As a result, market characteristics or policies that are able to reduce ambiguity can be very effective in improving market conditions.

Given the adverse effects of ambiguity on investor participation, securitization market liquidity, and the availability of credit in the real economy through bank lending, the model can be used to address the role of policy interventions in ameliorating these issues. As the model shows, risk retention requirements, such as the Dodd-Frank requirement that securitizers retain 5% of the credit risk of their securitized assets, are insufficient to address ambiguity concerns. Rather, effective policies must directly reduce ambiguity for investors. A prime example of such a policy is the Term Asset-Backed Securities Loan Facility (TALF), which offered loans to investors in the ABS market during the financial crisis from early 2009 through mid-2010. Since these loans were nonrecourse, the program essentially put a floor on the amount of losses an investor could face. This improved the worst-case outcome considered by investors, thereby reducing the amount of ambiguity. TALF was instrumental in both increasing ABS issuance and decreasing spreads as investors began to participate in the market again (Nelson (2011), Campbell, Covitz, Nelson, and Pence (2011)). Given the distinct equilibria that can arise as a result of ambiguity, policies that reduce ambiguity can have a positive impact on both origination in the real economy and participation in the financial markets, provided they shift ambiguity enough to change the market equilibrium.

This paper is related to three areas of the existing literature. First, it adds to the literature on limited market participation. Several explanations have been proposed to explain limited participation. Numerous studies, beginning with Allen and Gale (1994), have shown that a fixed participation cost can prevent full participation and can lead to a magnified impact of a small liquidity shock. However, models that are based on costs that are constant over time cannot explain the sudden change in market participation seen during the financial crisis. Alternatively, recent studies, including, among others, Cao et al. (2005), Garlappi et al. (2007), Epstein and Schneider (2007), Easley and O'Hara (2009, 2010), and Guidolin and Rinaldi (2010, 2013), have shown that ambiguity aversion can cause limited participation. The current model adds to this group of papers by allowing the degree of ambiguity to vary across states, which helps explain the sudden shift in participation seen in the securitization markets during the recent crisis.

Second, the model contributes to the literature on market freezes, specifically in the context of securitization markets.[3] There are several existing explanations of why market freezes can occur, including adverse selection, regulatory arbitrage, and neglected risks. These theories are examined in detail by Tirole (2011) and Leitner (2011) and will be discussed further in Section 5. An alternative explanation that is presented herein is ambiguity aversion, which is not mutually exclusive of these preceding theories, but serves to capture some features of the crisis that were not previously addressed. Uhlig (2010), which also looks at ambiguity aversion in the context of securitization, argues that ambiguity aversion can capture features of the crisis that adverse selection cannot. In particular in that model, under ambiguity aversion, a larger market share of distressed institutions can lead to a systemic bank run, as was seen during the crisis, while adverse selection predicts the opposite. Similarly, my model explains other features of the crisis that the alternative theories do not. Specifically, the model can explain both the timing and duration of the market freeze seen during the crisis. Additionally, it examines the securitization decision of the bank more specifically to understand how the bank's retention decision is made. Lastly, my model explains the freeze both in the financial markets and in lending in the real economy.

Lastly, the model contributes to understanding the link between financial intermediation (banks' securitization decisions) and the real economy (banks' loan origination decisions). Of

[3] This is also closely related to the literature on fire sales. See, among others, Coval and Stafford (2007), Diamond and Rajan (2011), and Shleifer and Vishny (2010a).

particular interest is how limited market participation of investors due to ambiguity can influence the number of loans intermediaries are willing to finance. The prevailing literature suggests that securitization will increase investment both by providing a source of funding and by allowing intermediaries to initiate loans but then securitize and sell them, reducing their own risk exposure.[4] However, in the case of a downturn, banks may reduce loan origination. Shleifer and Vishny (2010b) show that, when prices are influenced by investor sentiment, banks may stop originating in downturns, in favor of buying depressed securities. Similarly, this paper demonstrates that the origination benefits of securitization may also be reduced ex ante if there is uncertainty over whether the securities can in fact be sold.

The remainder of the paper is organized as follows. Section 2 presents a model of loan origination and securitization by banks in an economy with risk and ambiguity averse investors. I solve for the optimal demand functions of investors, which are then used to solve for the banks' optimal origination and retention decisions. Section 3 characterizes the equilibrium and discusses the impacts of key parameters, including the level of ambiguity, the return variance, and the expected payoff of loans, on the banks' optimal decisions. Section 4 considers the policy implications of the model, including a discussion of the impact of TALF. Section 5 discusses this model in relation to other existing theories, as well as the model's empirical implications. Section 6 concludes.

2. The Model

2.1. The Economic Environment

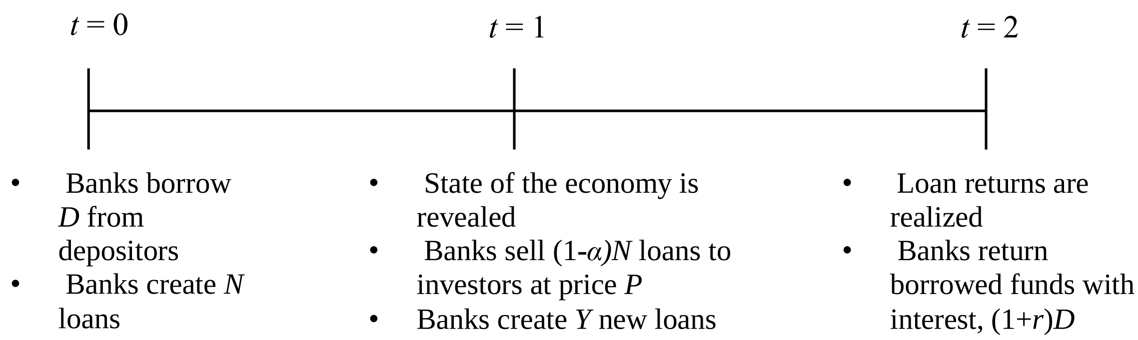
I consider a model of financial intermediation with securitization. In the model, banks can originate risky loans for a cost of \$1. These loans have a final payoff of V which is normally distributed with mean $\hat{V} > 1$ and variance σ^2. All loans are homogeneous. There is an infinite supply of loans that can be undertaken so banks are only limited in the number of loans they can originate by their funding. In order to differentiate between these positive NPV loans and those with $\hat{V} < 1$, an informed intermediary is necessary to screen and monitor the loans. Therefore, only banks can originate new loans.

[4] See, among others, Shin (2009), Nadauld and Sherlund (2013), and Shleifer and Vishny (2010b).

There are three types of participants in the model – banks, depositors, and investors. Banks can either hold cash or originate loans using their equity and money borrowed from depositors. Depositors receive a riskless return of $r \geq 0$ after loan payoffs are realized. After a bank has originated new loans, it can either hold the loans itself or securitize and sell them. If the bank chooses to securitize its loans, it also chooses the fraction of the loans it retains. The remainder is sold to risk and ambiguity averse investors. Using the cash on hand after securitizing, the bank can invest in new additional loans. These loans are essentially the same as the loans available initially in that they represent loans to individuals or firms. However, these loans will be modeled much more simply for the purpose of tractability. Specifically, banks can invest Y in loans that have an expected return of $i \in [0, i_{max}]$. Additionally, these loans cannot be securitized and pay off at the same time as the initial loans. The presence of these additional investment opportunities is the primary driver of securitization in the model.[5] The bank chooses to either retain or securitize loans strictly contingent on the relative payoffs of the two options, given the price impact of selling more securities and the benefits of additional investments. The role of adverse selection and the "skin in the game" benefits of retention, as discussed by Demiroglu and James (2012), and the use of securitization for regulatory arbitrage of capital requirements are not modeled here.

In this model, there are three time periods. At time 0, banks borrow money from depositors and originate loans. There are two possible states of the world and uncertainty over the state is resolved at the beginning of time 1. Banks then choose to retain the loans on their books or securitize and sell them to investors. With the cash on hand at time 1, banks can invest in another round of loans. All loan payoffs are realized and depositors are repaid at time 2. The timing of the model is shown in the figure below.

[5] Previous research suggests that funding is a primary driver of securitization, especially in nonmortgage ABS. See, among others, Ashcraft et al. (2012) and Loutskina (2011).



2.2. Investors

There are two possible states of the world. The only difference between these states is the degree of ambiguity that investors face. I develop the model with two states to capture the sudden shift in participation seen during the financial crisis. Intuitively, it is likely easier for investors to assess the potential payoffs of assets during normal times. However, in bad times, there is increased uncertainty, both idiosyncratically about the tail risks of securities themselves and systematically about the potential length and depth of the downturn, as well as possible government interventions. This increased ambiguity makes valuing assets more difficult for investors. The two-state design of the model captures the shift in uncertainty when a market downturn is severe.

In the model, there exists a pool of risk and ambiguity averse investors who can opt to buy the securities that the banks have originated. Depending on the state of the world, the investors have a given set of information about the payoff of the securities. The state is either high or low, denoted by $S \in \{H, L\}$, and the probability of the high state is $\Pr[S = H] = \theta$. In state H, there is no information asymmetry between banks and investors, and investors know that the expected value of a security is $\hat{V}$. In state L, investors face ambiguity about the expected value of the final payoff, V.[6] Specifically, they know $\mathrm{E}[V] \in [\underline{V}, \overline{V}]$, with $\underline{V} < \hat{V}$.[7] Although banks know the true expected payoff, $\hat{V}$, they cannot credibly convey this information to investors in state L.[8]

All investors have constant absolute risk aversion (CARA) utility of their final wealth $u(w) = -\exp(-\gamma w)$. Given the assumptions of CARA utility and normally distributed returns, each investor maximizes the utility function

$$(V - P)x - \frac{\gamma}{2}\sigma^2 x^2 \tag{1}$$

where P is the price at which investors can buy securities, x is the investor's demand for securities, and γ is the risk aversion coefficient. To account for investors' uncertainty over the value of the expected payoff, investor preferences can be modeled according to Gilboa and Schmeidler (1989), in which case investors maximize utility given the worst-case scenario. In this representation, investors have a set of priors, rather than a single prior, and use a max-min decision rule in determining their optimal solution. Therefore, the investor's maximization problem can be represented as

$$\max_x \min_{V \in [\underline{V}, \overline{V}]} (V - P)x - \frac{\gamma}{2}\sigma^2 x^2 \tag{2}$$

Solving the inner minimization reduces the problem to

$$\max_{x_H} (\hat{V} - P)x_H - \frac{\gamma}{2}\sigma^2 {x_H}^2 \tag{3}$$

in state H and, in state L,

$$\max_{x_L} (\underline{V} - P)x_L - \frac{\gamma}{2}\sigma^2 {x_L}^2 \tag{4}$$

This maximization problem gives the optimal demand for securities in states H and L, respectively, as

$$x_H = \begin{cases} \dfrac{\hat{V} - P}{\gamma\sigma^2} & \text{if } \hat{V} - P \geq 0 \\ 0 & \text{if } \hat{V} - P < 0 \end{cases} \tag{5}$$

and

[6] The model could have instead had investors facing ambiguity about the return variance, σ^2, with qualitatively similar results. The expected return was chosen to aid in the differentiation between risk and ambiguity in Section 3.4.

[7] There is no restriction on the ranking of $\overline{V}$ and $\hat{V}$.

[8] This may be because investors have full information, yet they are unable to fully interpret it given limitations in skill or time. Additionally, investors may rely on ratings to help them assess the potential value of securities, but given the decreased reliability of ratings during the crisis, may no longer be willing to do so.

$$x_L = \begin{cases} \dfrac{\underline{V} - P}{\gamma\sigma^2} & \text{if } \underline{V} - P \geq 0 \\ 0 & \text{if } \underline{V} - P < 0 \end{cases} \tag{6}$$

Since investors can buy but not sell securities in this setting, their demand is bounded at zero. Therefore, if the difference between their belief of expected value and price is less than zero, investors will opt not to participate. Given their ambiguity over expected value, investors will only purchase securities if their expected return is positive in the worst-case scenario.

There are several possible reasons that limited investor participation could be detrimental for banks. First, securitization serves as a funding source and enables banks to originate more loans than they would otherwise. By securitizing and selling their loans, they can then use their proceeds to originate new loans. If they cannot sell their securities due to limited participation, they will have to forego these other opportunities. Ambiguity among investors can reduce the ability for banks to securitize their loans, thereby reducing their profit as well as real economic activity. Banks must also have enough cash to repay their debt. Securitizing loans provides income that can be used to do so. If investors are unwilling to participate due to ambiguity, banks may be unable to repay their debts to depositors. These two issues are the primary focus of the model.

However, there are other explanations that are not modeled here. For instance, the loans that banks originate may be risky and they may want to sell them to avoid holding that risk. If securities cannot be sold due to limited participation, banks will be forced to hold more risk than they prefer. This issue is not at play in this model since it is assumed that banks are risk neutral. Another concern is that banks may need to sell a certain amount of their loans in order to meet capital requirements. Ambiguity aversion on the part of investors may force them to sell their assets at fire sale prices to make sure they meet these requirements. Although the current model does not have a capital constraint, the impact of a capital constraint is qualitatively very similar to the impact of the debt constraint modeled here. These other factors would increase the role that ambiguity plays in affecting both the banks' origination and securitization decisions, suggesting that the actual impact of ambiguity is even greater than that shown here.

2.3. Banks

Banks are risk neutral and know the true value of $\hat{V}$. They are endowed with some initial equity, E_0. Additionally, banks can raise cash, D, from depositors, paying a riskless return of $r \geq 0$ at time 2. Banks can either hold cash or originate loans. Let N be the number of loans the bank undertakes. If a bank has originated new loans, it can either hold them itself or securitize and sell them. The bank chooses the fraction α of the loans to retain; the remaining $1 - \alpha$ is sold to investors. Using the cash on hand after securitizing, the bank can invest Y in new loans at time 1.

I model a single non-competitive bank given that each bank in the real world issues unique securitization tranches that are not perfect substitutes. The bank's objective is to maximize profit, as specified by

$$\max_{N,\alpha,D} (1-\alpha)NP + \alpha NV - N - rD + iY \tag{7}$$

Here, the first term is the profit from the portion of loans that is securitized and sold to investors; the second term is the profit from the portion that is retained; the third term is the cost of originating the loans; the fourth term is the cost of repaying depositors; and the final term is the profit from additional loans originated at time 1.

Price is derived from the investors' inverse demand functions from equations (5) and (6) as $P = V - x\gamma\sigma^2$. Since investor demand must equal the supply of securities offered by the bank, $x = (1-\alpha)N$. Therefore, in equation (7), $P = V - (1-\alpha)N\gamma\sigma^2$.

While maximizing its expected profit in equation (7), the bank is subject to several constraints. The first constraint is a budget constraint at time 0. It requires the cost of initiating loans to be less than or equal to the funds available to the bank, which come from its equity and deposits. Specifically:

$$N \leq E_0 + D \tag{8}$$

The bank is also subject to a budget constraint when creating loans at time 1. Specifically, the amount invested in new loans, Y, cannot be more than the cash available from selling securities.

$$Y \leq (1-\alpha)N(V - (1-\alpha)N\gamma\sigma^2) \tag{9}$$

The second constraint imposes that the bank must be able to repay depositors with cash available at time 2. To ensure that depositors will always be repaid in full, banks cannot use the risky payoff from loans made at either time 0 or time 1 to repay depositors. Only their equity and profit from selling securities at time 1 can be used. While profit from selling securities at time 1 is, at that point, used to invest in new loans (Y), these loans will return at least Y at time 2, given that their return $i \in [0, i_{max}]$. This implies that:

$$(1+r)D \leq E_0 + D - N + (1-\alpha)N(V-(1-\alpha)N\gamma\sigma^2) \tag{10}$$

Next, the bank is subject to some minimum retention requirement. That is, the bank must retain at least $\underline{\alpha}$ of the loans it has originated.[9] Additionally, α is constrained from above by 1 since it is a fraction. Therefore:

$$\alpha \in \left[\underline{\alpha}, 1\right], 0 \leq \underline{\alpha} < 1 \tag{11}$$

Lastly, the price of the security must be non-negative[10], namely:

$$P = V - (1-\alpha)N\gamma\sigma^2 \geq 0 \tag{12}$$

Before proceeding, it is useful to make two observations. First, the bank will always choose to originate as many new loans as possible at time 1. That is, equation (9) is always binding. Given that loans have a non-negative return ($i \in [0, i_{max}]$) while holding cash has a zero return, the bank will invest any available cash in new loans.

Second, the bank's budget constraint at time 0 (equation (8)) will always bind before the riskless debt constraint (equation (10)). If the budget constraint does not bind, the bank will choose $D = 0$, given that there is no need to take on potentially costly deposits if the bank is not even using all of its equity, E_0. Therefore, the debt constraint becomes

$$0 = E_0 - N + (1-\alpha)N(V-(1-\alpha)N\gamma\sigma^2) \tag{13}$$

Since $E_0 - N > 0$ when (8) is not binding, $(1-\alpha)N(V-(1-\alpha)N\gamma\sigma^2)$ must be less than zero for (13) to hold. This would require $\alpha > 1$, which is a violation of (11). Therefore, (8) binds before (10). By extension, provided $\hat{V} > 1$, (8) is always binding since, if creating loans is profitable, the bank will originate as many as possible subject to its funding.

2.4. The Case with No Ambiguity

As a reference point, I first look at the case in which there is no ambiguity. That is, investors know the true value of the expected payoff, $\hat{V}$, in all states. The bank's maximization problem in this case is given by

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[9] This minimum threshold on α captures regulatory constraints that require banks to retain a minimum amount of the securities they create. For example, Section 941 of the Dodd-Frank Wall Street Reform and Consumer Protection Act, which was signed into law on July 21, 2010, requires securitizers to retain at least 5% of the credit risk of their securitized assets. While this requirement is meant to align the incentives of banks and investors, which is not a factor in this model, I include this constraint to show how such a regulation does not address the issues caused by ambiguity, as will be seen below.

[10] The price at which securities are sold can be less than 1, the cost of originating the underlying loans, if the bank must sell securities to meet its debt constraint or if the benefit of additional investment opportunities (i) outweighs the loss on the securities sold.

$$\max_{N,\alpha,D} \ (1+i)(1-\alpha)N(\hat{V}-(1-\alpha)N\gamma\sigma^2)+\alpha NV-N-rD \quad (14)$$

The bank is subject to the constraints in equations (8) – (12). Proposition 1 characterizes the bank's optimal decision in the case when $\hat{V}>1+r$. This condition requires that the return of a loan is greater than the cost of deposits. This simplification seems intuitive since it is likely that riskless deposits will have a lower return than risky loans. However, for completeness, the case in which $\hat{V}\leq 1+r$ is discussed in the appendix.

Proposition 1. *If $\hat{V}>1+r$, the bank's optimal decision without ambiguity, $\{N_{NA},\alpha_{NA},D_{NA}\}$, is defined as follows. The bank will borrow as much as possible, subject to its binding debt constraint. The bank retains $\alpha_{NA}=\max\left\{1-\frac{\hat{V}-\sqrt{\hat{V}^2-4\gamma\sigma^2(1+r)(N-E_0)}}{2N\gamma\sigma^2},\underline{\alpha}\right\}$. The optimal origination decision is given by $N_{NA}=\min\left\{E_0+\frac{\left(\hat{V}+i(1+r)\right)^2\hat{V}^2-\left(\hat{V}(1+r)\right)^2}{4\gamma\sigma^2(1+r)\left(\hat{V}+i(1+r)\right)^2},N\left(\underline{\alpha}\right)\right\}$. The bank's budget constraint is binding so $D_{NA}=N_{NA}-E_0$.*

Proof. See the Appendix.

As the return to originating a new loan, $\hat{V}$, increases, the number of loans originated will increase. This increase in origination is made possible by the accompanying increase in the amount of loans that are sold to investors as securities. Given that the return on loans is greater than the cost of deposits, the bank will originate as many loans as its debt constraint (equation (10)) will allow. This constraint is always binding when $\hat{V}>1+r$, as indicated in Proposition 1, given that the bank is borrowing more to take advantage of the higher profitability of loans.

2.5. The Case with Ambiguity

When ambiguity is present, investors make their decision based on the worst-case outcome, as described above. There are now two states, one in which investors know the true expected value and the other in which they only know a range of possible expected values. Therefore, the originator must determine a securitization strategy for both potential states. The origination decision, however, is made at time 0 before the state is known, so will take both states into account. The bank is again constrained to have its retained assets be less than or equal to its equity in both states, its costs less than its funds, its fraction of retained assets above the minimum threshold ($\underline{\alpha}$) and less than or equal to 1, and non-negative prices in both states. Therefore, the bank's maximization problem is

$$
\begin{aligned}
\max_{N,\alpha_H,\alpha_L,D} \quad & \theta\left[\,(1+i)(1-\alpha_H)NP_H + \alpha_H N\hat{V}\right] \\
& \qquad + (1-\theta)\left[\,(1+i)(1-\alpha_L)NP_L + \alpha_L N\hat{V}\right] - N - rD
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\text{subject to} \quad & N \le E_0 + D \\
& (1+r)D \le E_0 + D - N + (1-\alpha_H)NP_H \\
& (1+r)D \le E_0 + D - N + (1-\alpha_L)NP_L \\
& \alpha_S \in [\underline{\alpha}, 1]\ \forall\, S \in \{H, L\}, 0 \le \underline{\alpha} < 1 \\
& P_H = \hat{V} - (1-\alpha_H)N\gamma\sigma^2 \ge 0 \\
& P_L = \underline{V} - (1-\alpha_L)N\gamma\sigma^2 \ge 0
\end{aligned}
$$

The following proposition characterizes the bank's optimal decision under ambiguity. Again I focus on the case in which $\hat{V} > 1 + r$.

Proposition 2. *If $\hat{V} > 1 + r$, the bank's optimal decision under ambiguity, $\{N, \alpha_H, \alpha_L, D\}$, is defined as follows. The bank's optimal origination decision is $N \le N_{NA}$. The bank's budget constraint is binding so the bank borrows $D = N - E_0$. The bank's debt constraint is always binding in state L. Therefore, the bank retains:*

$$\alpha_L = \max\left\{1 - \frac{\underline{V} - \sqrt{\underline{V}^2 - 4\gamma\sigma^2(1+r)(N-E_0)}}{2N\gamma\sigma^2}, \underline{\alpha}\right\}$$

There exists a threshold $\underline{V}^*$ such that the bank's optimal retention decision in state H is given by:

(a) If $\underline{V} < \underline{V}^*$, the bank's debt constraint is slack in state H and the bank retains:

$$\alpha_H = \max\left\{1 - \frac{i\hat{V}}{2(1+i)N\gamma\sigma^2}, \underline{\alpha}\right\}$$

(b) If $\underline{V} \ge \underline{V}^*$, the bank's debt constraint is binding in state H and the bank retains:

$$\alpha_H = \max\left\{1 - \frac{\hat{V} - \sqrt{\hat{V}^2 - 4\gamma\sigma^2(1+r)(N-E_0)}}{2N\gamma\sigma^2}, \underline{\alpha}\right\}$$

Proof. See the Appendix.

The threshold $\underline{V}^*$ determines whether or not the bank's debt constraint binds in state *H*. If $\underline{V} < \underline{V}^*$, ambiguity is high. Therefore, the bank opts to originate fewer loans given its exposure to ambiguity averse investors in state *L*. Fewer deposits are required to fund these loans so the debt constraint in state H does not bind. However, when $\underline{V} \geq \underline{V}^*$, ambiguity is not as severe and the bank will originate more loans. This requires more deposits and the bank's debt constraint will therefore bind in both states. When this constraint is binding, the bank will have to sell more securities in order to repay depositors. Therefore, α_H is always lower when the debt constraint binds than it would be otherwise.

As the return to originating a new loan increases, the number of loans originated will increase. However, the effect on the bank's retention decision will depend on model parameters. The bank faces several tradeoffs when choosing between securitization and retention. First, the bank must consider the relative payoff from keeping loans, which depends on $\hat{V}$, versus the payoff from selling securities, which depends on *i* and price. Additionally, securitization can allow the bank to originate more loans initially. The bank must weigh the benefit of having a larger volume of loans against the decrease in price encountered when trying to sell more securities. Finally, the bank faces a tradeoff between creating more loans and being constrained by its funding, potentially leading to selling at fire sale prices if the state with ambiguity is realized, and creating fewer loans in order to be able to retain them all if the state with ambiguity is realized. Creating more loans may lead to more profits in the good state, but also increases exposure to the ambiguity averse investors in the bad state. These tradeoffs will be discussed in more depth in the following section.

3. Characterization of the Bank's Optimal Decision

To understand the role of ambiguity in determining the bank's optimal strategy, the origination and securitization decisions can be compared in the case with ambiguity versus the case without ambiguity. I consider the impact of several key model parameters, including the level of ambiguity, the return variance, and the expected payoff of loans, on the bank's optimal decisions in equilibrium in turn. Each section is summarized by results on origination and retention.

3.1. Ambiguity

The bank's decision to originate and securitize is dependent on the level of ambiguity in the market, as measured by $\underline{V}$. The lower $\underline{V}$ is relative to $\hat{V}$, the expected payoff of the security, the more ambiguity there is in the market. When $\underline{V}$ is very low and there is consequently a significant amount of ambiguity facing investors, the bank will choose to originate fewer loans so it will be able to retain them all when ambiguity is realized (state *L*). Given that investors' demand is based on the worst-case estimation of value, the price will be very low in this state. Therefore, the bank will be better off holding the securities itself relative to selling at a price significantly below the true expected value. On the other hand, when $\underline{V}$ is very close to $\hat{V}$ and there is, therefore, very little ambiguity in the market, the bank's optimal origination and securitization decisions will approach the equilibrium values under no ambiguity.

Figure 3 shows the effect of changing $\underline{V}$ on the bank's optimal decision for certain model parameters. Specifically, the parameters are fixed as follows: $E_0 = 10; \hat{V} = 1.2;\ i = 0.2;\ r = 0.05;\ \theta = 0.9;\ \sigma^2 = 0.1;\ \gamma = 1; \underline{\alpha} = 0.05$.[11] $\underline{V}$ ranges from 0 (extreme ambiguity) to $\hat{V}$ (no ambiguity). For small values of $\underline{V}$, the bank originates few loans to be able to retain all of them in state *L*. Therefore, even the threat of ambiguity can significantly reduce origination ex ante. The price in this state is too low to justify selling any securities, given the high level of ambiguity. However, the bank will sell as many securities as possible in state *H*. This is because the lower quantity of loans originated will result in a higher price, given investors' downward slopping demand curve.

As $\underline{V}$ increases, the number of loans the bank originates increases and the fraction that it retains in state *L* decreases. The bank's debt constraint is binding in state *L*, but as the price in *L* increases due to ambiguity decreasing, the bank can take on more deposits to originate more loans. In state *H*, decreasing ambiguity leads the bank to retain a higher fraction of its securities. However, as the bank is also increasing its origination, the total amount sold, $((1 - \alpha_H)N)$, remains constant and, correspondingly, so does the price in *H*. The fraction retained in state *L* eventually becomes lower than the fraction retained in state *H*, despite the lower price available

[11] These parameters are used only for explanatory purposes. The qualitative results are robust to a large range of potential parameters.
The return on loans originated at time 0 and additional investments originated at time 1 are assumed to be the same since they both represent loans to firms or consumers.
I assume the return on deposits, *r*, is 0.05 in my characterization of the equilibrium. However, the results would be qualitatively similar if $r = 0$ and banks therefore had unlimited funding.
I assume the minimum retention requirement, $\underline{\alpha}$, is 0.05 in accordance with the Dodd-Frank requirement that securitizers retain at least 5% of the credit risk of their securitized assets. However, this does not qualitatively affect the results; the minimum retention requirement could be removed entirely and similar results would be obtained. It is important to note that, while such a requirement may help with adverse selection, it does not alleviate the negative implications of ambiguity.

in state L. This is because, as $\underline{V}$ increases, the bank can originate more loans given the improving conditions in state L, but the bank must sell more of them in state L to meet its debt constraint.

As $\underline{V}$ gets even larger, the bank's debt constraint is now binding in both states H and L. This leads the bank to slow its increasing origination and begin selling more securities in state H to meet its constraint. The bank can now retain more securities in state L, decreasing its exposure to the ambiguity averse investors. As a result, the price falls in state H and rises in state L. As ambiguity continues to decrease, the equilibrium decisions of the bank approach those in the case with no ambiguity.

As with ex ante origination, the presence of ambiguity leads to lower investment at time 1 (bottom right panel). However, while this impact is relatively small in state H, it is very large in state L. This is because, even as the bank begins to sell more of its securities in state L, it is selling these securities at very low prices and so still has little cash with which to invest in new loans. As a result, policies that reduce ambiguity could have a particularly significant impact on investment in high ambiguity states, provided ambiguity can be reduced enough to change the equilibrium outcome.

Result 1a: For all levels of ambiguity, origination at times 0 and 1 are nondecreasing.
Result 1b: When ambiguity is high, the bank sells as much as possible in state H and retains everything in state L. As ambiguity decreases, retention in state H increases and retention in state L decreases. As ambiguity decreases further, the bank's debt constraint in state H becomes binding and these trends reverse.

These results are qualitatively very similar to the results for varying levels of state probability, which is also a measure of ambiguity in the market since, in state H, investors do not face ambiguity over the expected payoff of securities. The role of ambiguity is influenced by both the severity ($\underline{V}$) and the frequency (θ) of ambiguity. The main discussion focuses on the severity since this aspect of ambiguity is more easily influenced, as will be discussed further in section 4.

3.2. Return Variance

The other main factor influencing investors in this model aside from ambiguity is risk, as measure by return variance. Given that investors are risk averse, their price function is inversely related to the variance of returns (σ^2). Therefore, the bank's profits for selling securities are higher when variance is lower. As a result, the number of loans the bank originates increases as σ^2 decreases. In addition, the fraction of loans that the bank retains will decrease as σ^2 decreases since the bank wants to take advantage of higher prices by selling more securities.

Figure shows the effect of changing σ^2 on the bank's optimal decision for certain model parameters. Specifically, the parameters are fixed as follows: $E_0 = 10; \hat{V} = 1.2;\ \underline{V} = 0.7;\ i = 0.2; r = 0.05;\ \theta = 0.9;\ \gamma = 1; \underline{\alpha} = 0.05;\ D_{max} = 100$. The presence of ambiguity decreases the number of loans originated for all values of σ^2 and increases the fraction of those loans retained for high values of σ^2. In this example, the bank's debt constraint is always binding in state L, so the bank will sell more securities to meet this requirement for low values of σ^2. However, for higher values of σ^2, in order to satisfy its debt constraint, the bank will also reduce origination. The bank consistently sells more securities in state L than in state H, despite the presence of ambiguity, due to its binding debt constraint.

The fraction retained is generally increasing in σ^2, while the amount of origination is decreasing. Since σ^2 directly influences the price investors are willing to pay, a higher σ^2 leads to lower prices, which means the bank will opt to retain more and will need to originate less in order to do so, given its debt constraint.

When the variance of returns is high, the fraction of securities retained approaches 1 in all cases.[12] This is due to the negative relationship between variance and prices. In these cases, since the bank is not selling much to investors, the impact of ambiguity on retention is relatively low. However, for small values of σ^2, the difference in retention between the state with ambiguity and the state without ambiguity becomes more significant. Notably though, the primary effect of ambiguity is on origination, especially when variance is low. The bank can originate many more loans without ambiguity. It is limited in its origination under ambiguity due to the debt constraint in state L. Therefore, decreasing ambiguity for securities with low variance can have a significant impact on origination in the real economy.

[12] This is a direct result of the assumption that banks are risk neutral and investors are risk averse. Therefore, as the risk of the loans increases, the banks are better able to bear this risk so they retain more of their loans.

Result 2a: For all levels of variance, origination at times 0 and 1 are nonincreasing.

Result 2b: When variance is low, the bank sells as much as possible in both states *H* and *L*. As variance increases, retention in both states increases, with retention in state *L* always less than retention in state *H* given the bank's debt constraint.

3.3. Expected Payoff of Loans

Both the return to retaining loans and the return to selling securities are directly tied to the expected payoff ($\hat{V}$) of the loans originated. Therefore, as the expected payoff increases, the bank chooses to originate more loans. The number of loans originated is nondecreasing in $\hat{V}$. However, the relationship between $\hat{V}$ and the fraction retained is not as direct. In both the case of no ambiguity and state *H* under ambiguity, an increase in $\hat{V}$ increases the profit to both retention and securitization. If the bank securitizes more, the increased supply of securities in the market will result in a lower price, which suggests the bank should retain more. However, by retaining more, the bank foregoes additional investment opportunities. Therefore, the impact of a higher $\hat{V}$ on retention depends on its impact on price relative to the return on additional investments (*i*). On the other hand, in state *L*, only the profit to retention is increased with an increase in $\hat{V}$. Changes in $\hat{V}$ do not affect the price ambiguity averse investors will pay since they are only concerned with the worst-case outcome, $\underline{V}$. As a result, when not bound by its debt constraint, the bank has the incentive to retain as much as possible in state *L*.

Figure shows the effect of changing $\hat{V}$ on the bank's optimal decision for certain model parameters. Specifically, the parameters are fixed as follows: $E_0 = 10$; $\underline{V} = 0.7$; $i = 0.2$; $r = 0.05$; $\theta = 0.9$; $\sigma^2 = 0.1$; $\gamma = 1$; $\underline{\alpha} = 0.05$. When $\hat{V} < 1$, the bank has no incentive to originate loans since the payoff ($\hat{V}$) does not exceed the cost (\$1). Therefore, the focus is on results for $\hat{V} \geq 1$. As expected, the number of loans originated is nondecreasing in $\hat{V}$ in cases both with and without ambiguity, and the number of loans originated with ambiguity is significantly less than the number originated without ambiguity. Under ambiguity, the number of loans the bank can originate is limited due to the bank's debt constraint. This constraint is binding in state *L*, so the bank sells more securities to meet it. However, once the bank reaches its minimal amount of retention ($\underline{\alpha}$), it cannot originate any more loans since it cannot sell more securities to meet its debt constraint. For lower values of $\hat{V}$, the bank opts to retain all of its loans in state *L*. However, as $\hat{V}$ increases, the bank retains a smaller fraction in state *L* because its debt constraint has become binding, so for higher values of $\hat{V}$, the amount retained in state *L* can again be less than the amount retained in state *H*. The bank trades off receiving a lower price due to selling more securities in state *L* for creating more loans and retaining more of them in state *H*.

In the case with no ambiguity, the large increase in retention for low values of $\hat{V}$ is due to the bank's debt constraint becoming binding. This occurs when $\hat{V}$ becomes larger than 1 + *r* and it therefore becomes profitable to take on deposits. Subsequently, as $\hat{V}$ increases, the bank sells enough securities to meet its debt constraint.

Result 3a: For all levels of expected payoff, origination at times 0 and 1 are nondecreasing.
Result 3b: When expected payoff is low, the bank sells most of its securities in state *H* and retains everything in state *L*. As expected payoff increases, the bank's debt constraint binds in state *L* so retention decreases. Meanwhile, retention in state *H* increases until $\alpha_L = \underline{\alpha}$. At this point, origination reaches its maximum so the bank can sell a larger fraction in state *H*.

3.4. Ambiguity vs. Risk

I now turn to a comparison of the impact of risk versus the impact of ambiguity by removing both ambiguity and risk from the model in turn. I start with a model with only risk and no ambiguity. In this case, investors know the true expected value of loans, $\hat{V}$, in both states. In other words, $\underline{V} = \hat{V}$. Therefore, the bank's optimization problem reduces to

$$\max_{N,\alpha,D} \;(1+i)(1-\alpha)N\left(\hat{V}-(1-\alpha)N\gamma\sigma^2\right)+\alpha N\hat{V}-N-rD \tag{16}$$

Alternatively, a setting with ambiguity and no risk is equivalent to $\sigma^2 = 0$. Therefore, the bank's optimization problem is

$$\begin{aligned}\max_{N,\alpha_H,\alpha_L,D} \;& \theta\left[\,(1+i)(1-\alpha_H)N\hat{V}+\alpha_H N\hat{V}\right] \\ & + (1-\theta)\left[\,(1+i)(1-\alpha_L)N\underline{V}+\alpha_L N\hat{V}\right]-N-rD\end{aligned} \tag{17}$$

An increase in either risk or ambiguity serves to decrease the price that investors are willing to pay for securities. However, the impact of ambiguity is more abrupt. Specifically, ambiguity affects price directly proportional to the change in ambiguity, as measured by a lower $\underline{V}$, since price is simply $\underline{V}$ in this case. On the other hand, when there is risk, price is $\hat{V} - (1-\alpha)N\gamma\sigma^2$. Therefore, increasing risk, as measured by a higher σ^2, not only impacts price directly through σ^2, but also indirectly through $(1-\alpha)N$. As risk increases, the negative impact on price can be counteracted with a simultaneous decrease in origination and/or securitization. As a result, the impact of ambiguity is more absolute than the impact of risk. Specifically, there is a threshold on $\underline{V}$, as defined in the following proposition, above which the bank retains as little as possible and below which the bank retains as much as possible.

Proposition 3. *When there is ambiguity but no risk in the market, the bank will always choose* $\alpha_H = \underline{\alpha}$*. There exists a threshold* V^* *such that:*

a) *If* $\underline{V} > V^*$*, ambiguity is low and the bank chooses* $\alpha_L = \underline{\alpha}$ *and* $N = \frac{(1+r)E_0}{1+r-(1-\alpha_L)\underline{V}}$.

b) *If* $\underline{V} \leq V^*$*, ambiguity is high and the bank chooses* $\alpha_L = 1$ *and* $N = E_0$.

Proof. See the Appendix.

Intuitively, this threshold V^* is the maximum amount of ambiguity the market can bear while remaining operational. If ambiguity is too large, the bank chooses not to sell any securities in the state with ambiguity given the depressed price it will face. A lower threshold is preferable since it will require more ambiguity to cause a market freeze. Considering the partial derivatives of V^* reveals that the threshold decreases when $\hat{V},\ \theta,$ or i increase and when $\underline{\alpha}$ or r decrease. All of these changes in model parameters make securitization more profitable and the bank will therefore be willing and able to securitize their loans under higher values of ambiguity.

The difference between risk and ambiguity can be seen in Figure , where the bank's origination and retention decisions are plotted under risk with no ambiguity and under ambiguity with no risk for certain model parameters. Specifically, the parameters are fixed as follows: $E_0 = 10; \hat{V} = 1.2;\ i = 0.2;\ r = 0.05;\ \theta = 0.5;\ \gamma = 1; \underline{\alpha} = 0.05$. In the first case with just risk, I set $\underline{V} = \hat{V}$ and consider $\sigma^2 \in [0,0.2]$. In the second case with just ambiguity, I set $\sigma^2 = 0$ and consider $\underline{V} \in [0, \hat{V}]$. I also impose a maximum on the amount of available deposits, D_{max}, to prevent origination from going to infinity as risk and ambiguity go to zero. I set $D_{max} = 100$. This is why origination is capped at $110 = E_0 + D_{max}$ in both the case of low risk and low ambiguity.

Under both very low risk and very low ambiguity, the bank will originate as many loans as possible and securitize them all. Conversely, under both very high risk and very high ambiguity, the bank will originate fewer loans and retain them all. However, ambiguity and risk differ significantly in the intermediate ranges. As discussed above, the negative impact of increased risk can be mitigated by lower origination or higher retention. Therefore, as shown in Figure , both outcome variables change gradually as risk increases.

On the other hand, in the case of ambiguity, the optimal decisions behave very differently, especially for retention. Specifically, there are two distinct regions in the plot of retention under ambiguity. There is not a gradual change between the extremes of full retention and full securitization, as there is in the case of risk. Once the level of ambiguity increases above the threshold, V^*, as defined in Proposition 3, retention immediately increases from $\underline{\alpha}$ to 1. Crossing this threshold also causes a drop in origination. Under the current model parameters, this drop is not very severe. However, when either E_0 or V^* is higher, the extremity of this fall in origination increases since origination is higher in this case.

The shift between these equilibria under ambiguity is very dramatic. Because of this, a small change in the level of ambiguity, especially around V^*, can have a large effect on both the origination and, in particular, the securitization decisions of the bank. Therefore, market characteristics or policies that are able to reduce ambiguity can be very effective in improving market conditions.

To add more realism to this setting, Figure shows the optimal decisions under varying degrees of both risk and ambiguity. As mentioned above, origination is highest when both risk and ambiguity are low. However, the decline is origination is much steeper as ambiguity increases than as risk increases.

With respect to retention in state L, the threshold defined in Proposition 3 can be seen clearly when risk is low. Retention jumps from minimal retention (blue) when ambiguity is low directly to full retention (red) when the threshold is crossed. However, as risk increases, this shift in retention becomes progressively more gradual. This suggests that ambiguity can have more drastic effects in low risk securities, while the impact for high risk securities is more tempered.

This is consistent with what occurred in securitization markets during the financial crisis. ABS markets froze abruptly in late 2008 but subsequently unfroze quickly with the initiation of TALF. Issuance volumes returned to near pre-crisis levels in auto, credit card, and student loan backed ABS by mid-2009 (see Figure 1). This is in contrast to the behavior of the higher risk

CMBS market, which gradually increased issuance after freezing in late 2008. However, as of early 2014, CMBS issuance still remains far below pre-crisis levels.

4. Policy Implications

As discussed in Section 3.4, the presence of ambiguity leads to two very distinct equilibria and small changes in ambiguity can have large changes in the outcome, especially among low risk securities. This model lends itself to consideration of policies that help alleviate the negative implications of ambiguity, including origination of fewer loans and limited participation in the market. Ambiguity can lead to market freezes even when retention requirements, such as those imposed by Dodd-Frank, are in place, as the model shows by requiring banks to hold at least $\underline{\alpha}$ of their securities. While these types of requirements are important for aligning the incentives of banks and investors, thereby reducing risk, they do not reduce ambiguity, leaving the market still vulnerable to freezes. An alternative solution would be to have an insurance-like contract, either from the private market or the government, which would protect investors from losses. The model shows that increasing the minimum expected payoff considered by investors can increase the price at which these investors are willing to participate. An insurance-like contract could implement this increase.

Consider the decision making process of an ambiguity averse investor choosing whether or not to participate in the ABS market. Given the complexity of these securities, the investor may not be able to determine their true expected payoffs. First, the investor may only have coarse information available given the opacity of these markets. Second, even if the information is revealed by the issuer, the investor may not have the ability, resources, or time to correctly interpret this information. Therefore, the investor will consider a range of outcomes, for some of which the payoff of the securities may be very low. Even if these outcomes are unlikely, an ambiguity averse investor is highly influenced by the worst-case scenario. Therefore, these investors are only willing to participate in the market at unrealistically low prices that account for these potential bad outcomes.

Now consider the case in which there exists some contract that can put a lower threshold on the expected payoffs considered by investors. A prime example of such a program is the Term Asset-Backed Securities Loan Facility (TALF).[13] The Federal Reserve announced TALF in

[13] For a complete description of TALF, see Ashcraft et al. (2012).

November 2008 to make loans to investors in eligible ABS in response to the drastic reductions in origination and trading in ABS markets during the financial crisis. TALF provided investors with nonrecourse loans with which to buy securities. Therefore, if the value of the securities purchased turned out to be very low, the investor did not have to repay the loan and could simply pass the security, and its resulting losses, on to the government instead. This effectively put a limit on how much investors could lose on their investments in asset-backed securities, which is equivalent to increasing $\underline{V}$ in the model.

In this case, the ambiguity averse investors are willing to participate in the market at a higher price closer to the securities' true expected value. Not only does this approach increase trading in the securitization market, it also increases origination among banks ex post by providing more access to the funding benefits of securitization. Once a program such as TALF is implemented, banks are better able to sell their securities and can then use those proceeds to originate new loans.

However, a government program like TALF is mainly beneficial in unfreezing markets that have already broken down; an alternative solution that could reduce ambiguity ex ante would be even more beneficial. An insurance-like contract that would pay investors when the value of the underlying collateral fell below a certain point would encourage ambiguity averse investors to participate in the market even in high ambiguity environments. Such a contract would be similar to a credit default swap except that the "default" event would be a decline in the asset value by some predetermined amount. This would not only have the ex post benefits of a government guarantee program, but it would also increase origination ex ante since banks would not have to prepare for the case when investors flee the market if ambiguity gets too high.

A program that reduces ambiguity for investors, whether through the government or through the private market, is analogous to a program like deposit insurance. Deposit insurance prevents depositors from running on a bank by insuring the full repayment of their deposits up to a certain value. Without such insurance, it may be rational for depositors to run if they suspect other depositors will run. Therefore, an illiquid bank can become insolvent due to the inherent mismatch in the durations of their assets and liabilities. Implementing deposit insurance averts this negative equilibrium by removing the reason for depositors to run.

Likewise, in the case of ambiguity aversion, investors rationally opt not to participate in a market unless the price of the security is lower than their worst-case expectation of its payoff. A program like TALF reduces ambiguity, thereby inducing investors to participate in the market. If non-participation is driven by ambiguity, as opposed to risk, then such a program should be costless to implement, similar to deposit insurance. In the case of deposit insurance, the presence of the program itself prevents bank runs, making it costless to insure illiquid banks. Likewise, a program like TALF encourages investors to purchase securities, which are not excessively risky securities but rather securities about which the investors are not sure they have full information. Therefore, it should be rare that a security will turn out to be low value and the investor will not repay their loan. This was in fact the case with TALF. As of March 2011, TALF had earned nearly $600 million in interest (Nelson (2011)), and there were not any losses on the loans, which have all subsequently been repaid.

While such programs may have the benefits of increasing participation and origination at a negligible cost, they also have the threat of increased moral hazard. Given the presence of an insurance-like contract, banks may be less diligent about the quality of loans they make since investors are more willing to participate even when the underlying securities are highly ambiguous or risky. Increasing moral hazard may be acceptable if ambiguity can be reduced enough to induce participation. However, this tradeoff must be considered before implementing such a program, especially since there are some regions in which a change in ambiguity will not affect the outcome at all and others in which a small change in ambiguity can have a large impact on the resulting equilibrium. However, the model cannot currently speak to this tradeoff. Moral hazard would increase the origination of negative NPV loans, but the model, as it stands, does not have heterogeneous loans.

5. Discussion

There are several other existing theories of market freezes in securitization markets, which can be compared to the model presented here. These include theories of adverse selection, regulatory arbitrage, and neglected risks. Gennaioli et al. (2013) discuss these theories and their relative strengths in detail. I will provide a brief overview of these theories here, then discuss the additional contribution of my model with ambiguity aversion.

First, many papers discuss the role of adverse selection in securitization. Financial intermediaries can pool and tranche assets to create informationally insensitive securities. Since all investors are symmetrically informed about these securities, the result is a highly liquid, low risk market. However, as bad news comes into the market, investors realize they may in fact be at an informational disadvantage and choose not to trade, resulting in a market freeze. Work in this area includes, among others, DeMarzo and Duffie (1990), Riddiough (1997), DeMarzo (2005), Pagano and Volpin (2012), and Dang et al. (2010).

The theory of regulatory arbitrage is described by, among others, Calomiris and Mason (2004), Leitner (2011), and Acharya et al. (2013). It suggests that banks can use securitization as a means to reduce capital requirements. By holding highly rated securitization tranches instead of the underlying assets or by holding securities in an off-balance sheet conduit, banks can increase leverage while still adhering to their capital requirements. However, when there is bad news and the prices of these assets decline, banks may not be able to sell these assets while still meeting their capital requirements. Thus, a market freeze can occur.

Recent work by Gennaioli et al. (2012, 2013) develops a third theory of shadow banking involving neglected risks. Investors ignore certain tail risks, leading to increased securitization and creation of debt that is perceived as risk free. However, when it is revealed that this debt is riskier than anticipated, intermediaries default on their obligations and a liquidity crisis can occur given the increased leverage allowed by securitization. Coval et al. (2009) also show that market freezes in securitization can result from imprecise models that underestimate or neglect certain risks. This theory reconciles the fact that, during the recent financial crisis, markets did not seem to fully understand the risks in securitization markets, even as bad news started to be revealed. Despite negative information on the housing market beginning to surface in mid-2007, securitization markets did not crash until late 2008. This crash was largely unanticipated by investors, as indicated by the spreads on these securities, which suggests that it was not just news about increased risks that froze markets. The neglected risks hypothesis is the only one of these prior theories that can account for this delay in the crash by suggesting that there were risks that investors were unaware of until late 2008.

However, the existing theories face some limitations in explaining the recent crisis. While my model is not mutually exclusive of these other theories, it does address some of these limitations. In particular, while these theories can explain how a liquidity crisis can occur, they do not explain how one can persist. Although the neglected risks hypothesis can explain the timing of the securitization market crash, it does not explain why the market took time to recover. It is unclear why, once these neglected risks are realized, the market should continue to falter. On the other hand, ambiguity aversion can explain both the timing and duration of the market freeze seen during the crisis. Given the presence of ambiguity, the market freeze can persist as long as investors remain uncertain about the expected outcome. This is particularly evident in the CDO markets, where origination is still low even several years after the initial crisis, despite limited losses on these securities.[14]

The model provides several empirical implications. First, banks will hold more securities during crises because they face prices much lower than the true value of their securities. It becomes more difficult to sell these securities but, since there is a lag between origination and securitization, banks will be forced to retain a greater portion of the loans they have already originated. This is particularly true when ambiguity is severe or when the returns to securitization are low. This prediction is supported by Erel et al. (2014), who show that the aggregate holdings of highly-rated tranches increased between the end of 2006 and the end of 2007. More specifically, the model suggests that, during times of heightened ambiguity, firms without binding funding constraints will choose to retain their securities and markets will freeze, while constrained firms will be forced to sell their securities at fire sale prices. The model also suggests that prices will be lower when ambiguity is high. However, the amount of securitization may be higher or lower, depending on whether or not the firm is constrained by its debt. Lastly, during times of heightened ambiguity, origination of new loans will be severely restricted, even when ambiguity is relatively mild.

The model's implications about ambiguity and risk also yield a number of empirical predictions. While the impact of risk on securitization is continuous, the impact of ambiguity is more disjointed. There are several distinct equilibria with respect to ambiguity when risk is low so a small change in the level of ambiguity can create a large change in the bank's origination and securitization decisions, as the economy shifts to a different equilibrium. This implies that, under ambiguity, market freezes can both start and end abruptly as the amount of ambiguity changes, especially among low risk securities. Additionally, as discussed in Section 4, in the presence of ambiguity, a program like TALF aimed at inducing participation will be costless. If the program were simply reducing the risk to investors, this would not be the case since the

[14] For example, only 32 tranches in 14 CLOs of all of the 4118 tranches in 719 US broadly syndicated arbitrage cash flow CLOs that Moody's has rated since 1996 have suffered principal losses at maturity (Xu 2012).

government would bear the remaining risk and would suffer losses on the poorly performing securities.

6. Conclusion

I present a model of financial intermediation in which securitization markets can freeze or at least experience a significant decrease in volume when investors are ambiguity averse. In cases where ambiguity is severe or frequent, or expected payoffs are low, securitization is significantly reduced when the state in which investors face ambiguity is realized. However, when ambiguity is mild or the returns to securitization are high, banks will opt to originate more loans. This may force banks to sell their loans at fire sale prices when the state with ambiguity is realized given that the banks' debt constraint is now binding. Therefore, the presence of ambiguity can explain both market freezes and fire sales. The potential for ambiguity aversion among investors also causes a decrease in the number of loans that banks are willing to undertake in all cases. As a result, origination can decrease not just ex post but also ex ante, and there are both financial and real economic consequences to ambiguity in securitization markets. Additionally, the presence of ambiguity averse investors can help explain both the timing and, in particular, the duration of the freeze in securitization markets seen during the 2008 financial crisis in a way that existing theories cannot.

Measures which reduce ambiguity, either by reducing the worst-case outcome considered by investors or by reducing the probability that the state in which ambiguity is present occurs, can ameliorate conditions in both the real economy and in the securitization markets. By reducing ambiguity ex ante, more loans will be undertaken by the bank, since there will be better prospects for selling those loans as securities. Ambiguity can also be reduced ex post once the state with ambiguity is realized. Improving the worst-case outcome considered by ambiguity averse investors can encourage banks to securitize their loans and revive trading in securitization markets. This can have the added benefit of freeing up capital with which banks can originate more new loans, in turn stimulating the real economy as well. Given the nature of ambiguity, small changes in the level of ambiguity can have large impacts on both origination and securitization outcomes.

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Figure 1. ABS Issuance

This figure shows the quarterly US issuance of asset-backed securities by class from 2008 to 2013. Source: Securities Industry and Financial Markets Association (SIFMA), [US ABS Issuance and Outstanding]() – issuance data from 1985 to April 2015

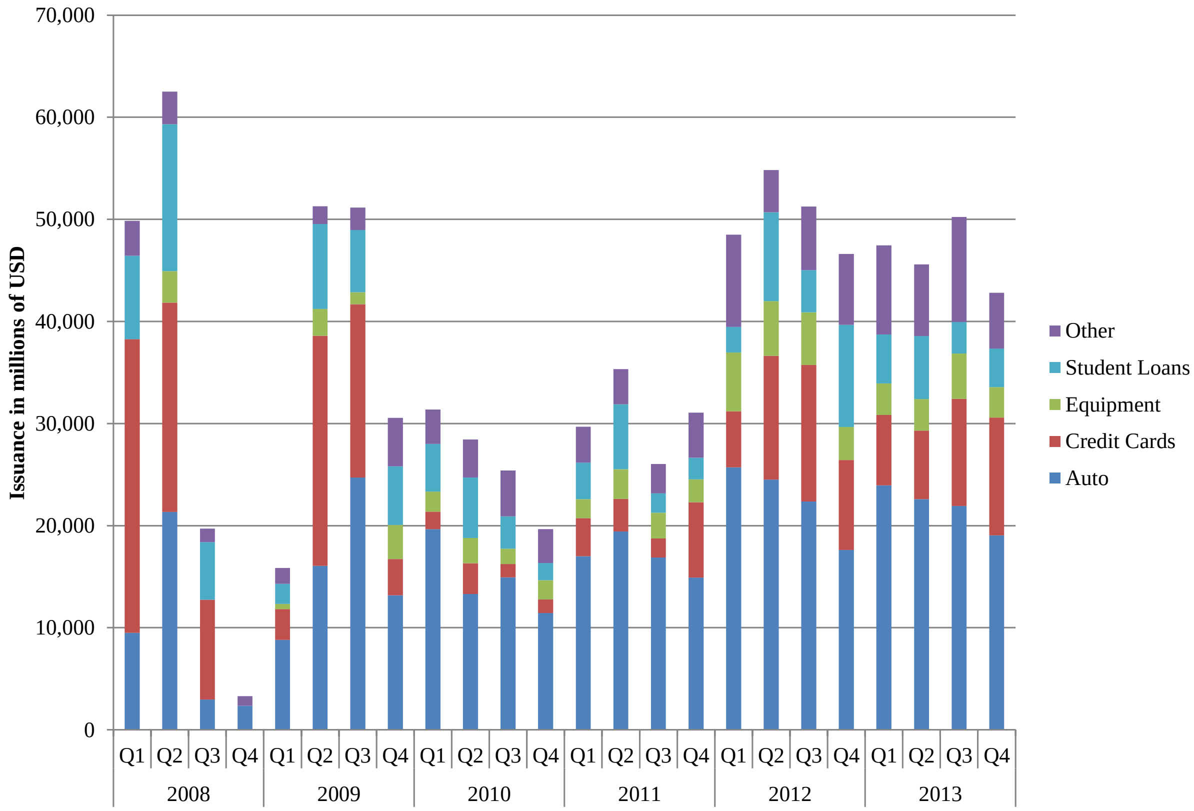


Figure 2. CDO Issuance

This figure shows the annual global issuance of collateralized debt obligations by collateral type from 2000 to 2013. CLOs are securities with corporate loans as collateral. CBOs are securities with high yield or investment grade corporate bonds as collateral. Structured Finance CDOs are securities with assets such as RMBS, CMBS, ABS, or other CDOs as collateral.
Source: Securities Industry and Financial Markets Association (SIFMA), Global CDO Issuance and Outstanding – issuance data from 2000 to 2015 Q1

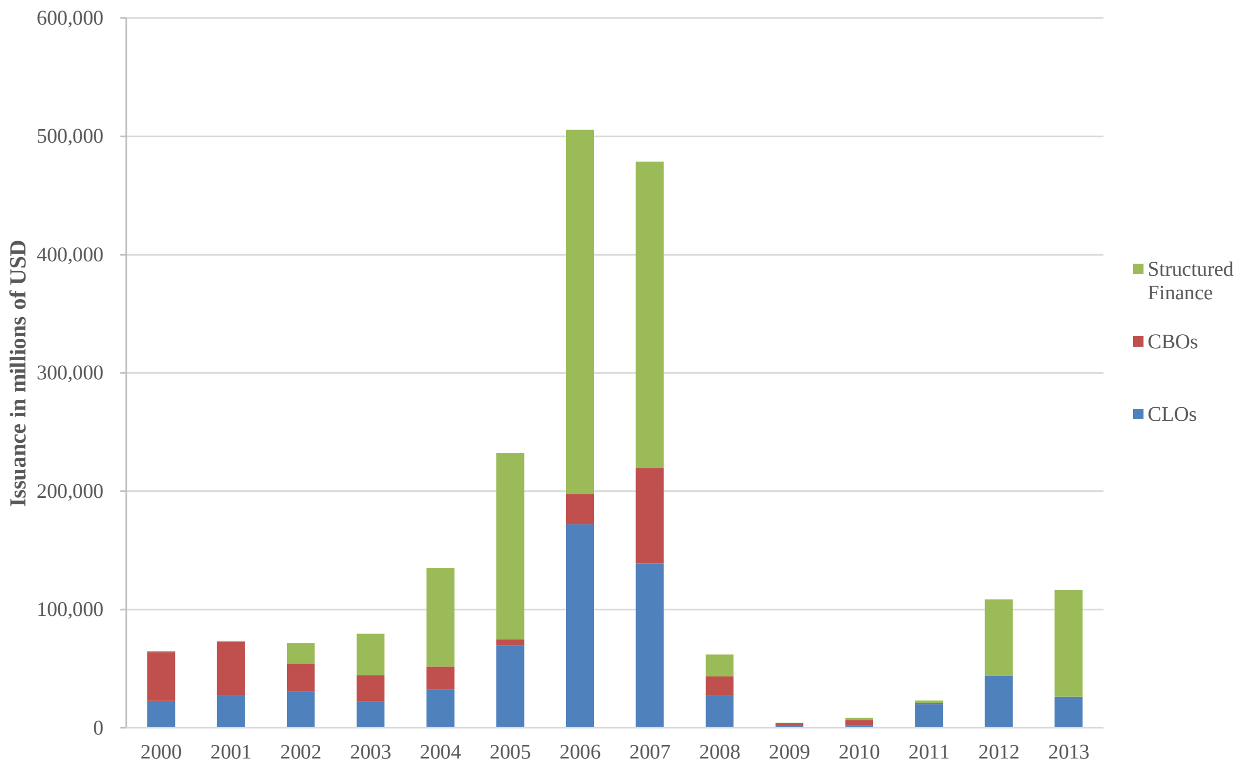


Figure 3. Bank's Decisions and Ambiguity

This figure shows how the bank's optimal origination and securitization decisions are affected by the level of ambiguity, as measured by $\underline{V}$. $\underline{V}$ ranges from 0 (extreme ambiguity) to $\hat{V}$ (no ambiguity). The solid line is the case with no ambiguity and the dashed line is the case with ambiguity. In the plots for retention, price, and additional investment, the dashed line is state H and the dotted line is state L. Model parameters are fixed as follows: $E_0 = 10; \hat{V} = 1.2;\ i = 0.2; r = 0.05;\ \theta = 0.9;\ \sigma^2 = 0.1;\ \gamma = 1; \underline{\alpha} = 0.05$.

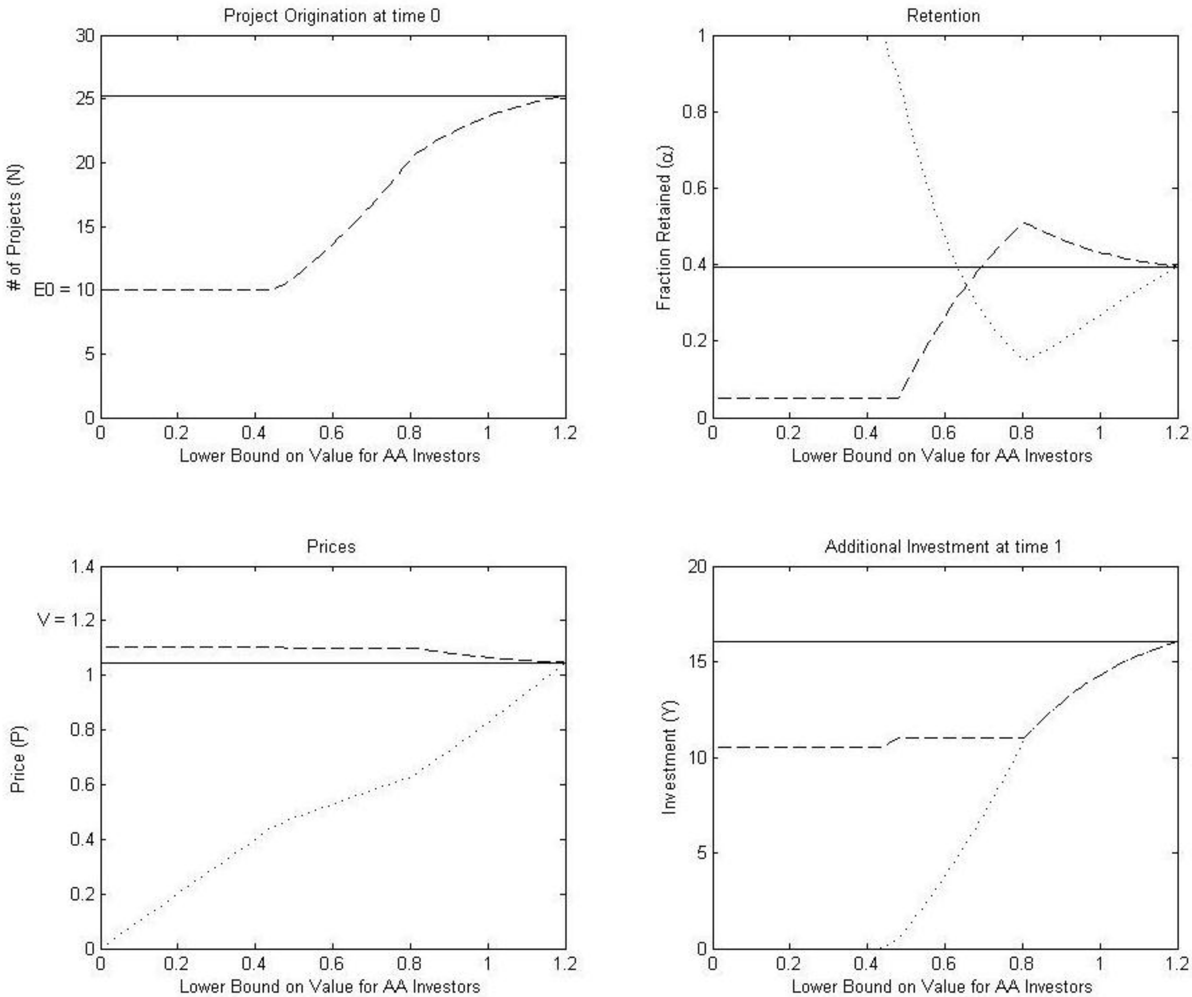


Figure 4. Bank's Decisions and Return Variance

This figure shows how the bank's optimal origination and securitization decisions are affected by the variance of returns (σ^2). The solid line is the case with no ambiguity and the dashed line is the case with ambiguity. In the plot of retention, the dashed line is state H and the dotted line is state L. Model parameters are fixed as follows: $E_0 = 10; \hat{V} = 1.2;\ \underline{V} = 0.7;\ i = 0.2;\ r = 0.05;\ \theta = 0.9;\ \gamma = 1; \underline{\alpha} = 0.05$.

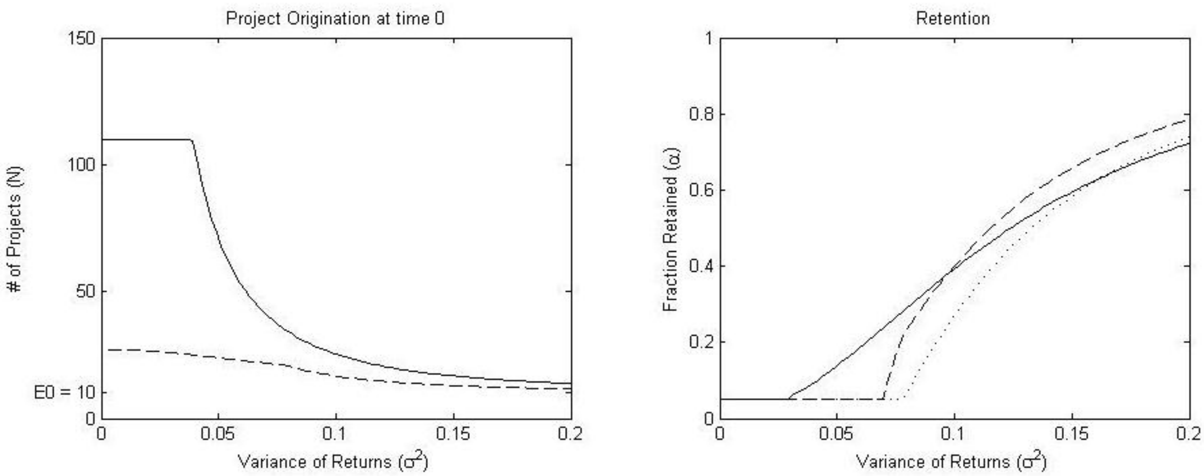


Figure 5. Bank's Decisions and Expected Payoff of Securities

This figure shows how the bank's optimal origination and securitization decisions are affected by the expected payoff of loans ($\hat{V}$). The solid line is the case with no ambiguity and the dashed line is the case with ambiguity. In the plot of retention, the dashed line is state H and the dotted line is state L. Model parameters are fixed as follows: $E_0 = 10$; $\underline{V} = 0.7$; $i = 0.2$; $r = 0.05$; $\theta = 0.9$; $\sigma^2 = 0.1$; $\gamma = 1$; $\underline{\alpha} = 0.05$.

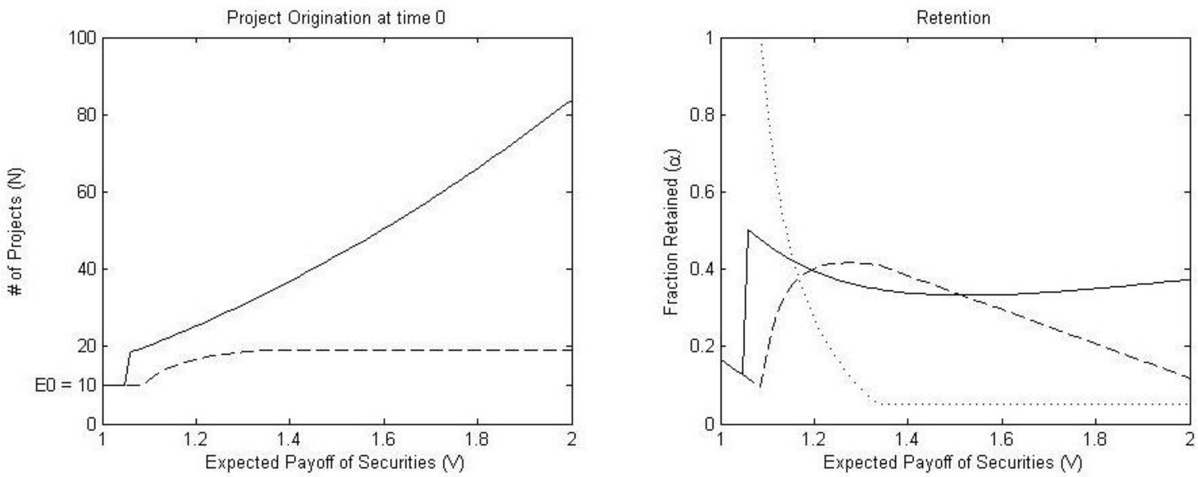


Figure 6. Ambiguity vs. Risk

This figure shows how the bank's optimal origination and securitization decisions are affected when there is risk but no ambiguity (top) versus when there is ambiguity and no risk (bottom). The plot of retention with ambiguity displays the retention in state L; retention in state H is always $\underline{\alpha}$. In the case with only risk, model parameters are fixed as follows: $E_0 = 10; \hat{V} = 1.2;\ \underline{V} = \hat{V};\ i = 0.2;\ r = 0.05;\ \theta = 0.5;\ \gamma = 1; \underline{\alpha} = 0.05; D_{max} = 100;$ and $\sigma^2 \in [0,0.2]$. In the case with only ambiguity, model parameters are fixed as follows: $E_0 = 10; \hat{V} = 1.2; i = 0.2;\ r = 0.05;\ \theta = 0.5;\ \sigma^2 = 0;\ \gamma = 1; \underline{\alpha} = 0.05; D_{max} = 100;$ and $\underline{V} \in [0, \hat{V}]$.

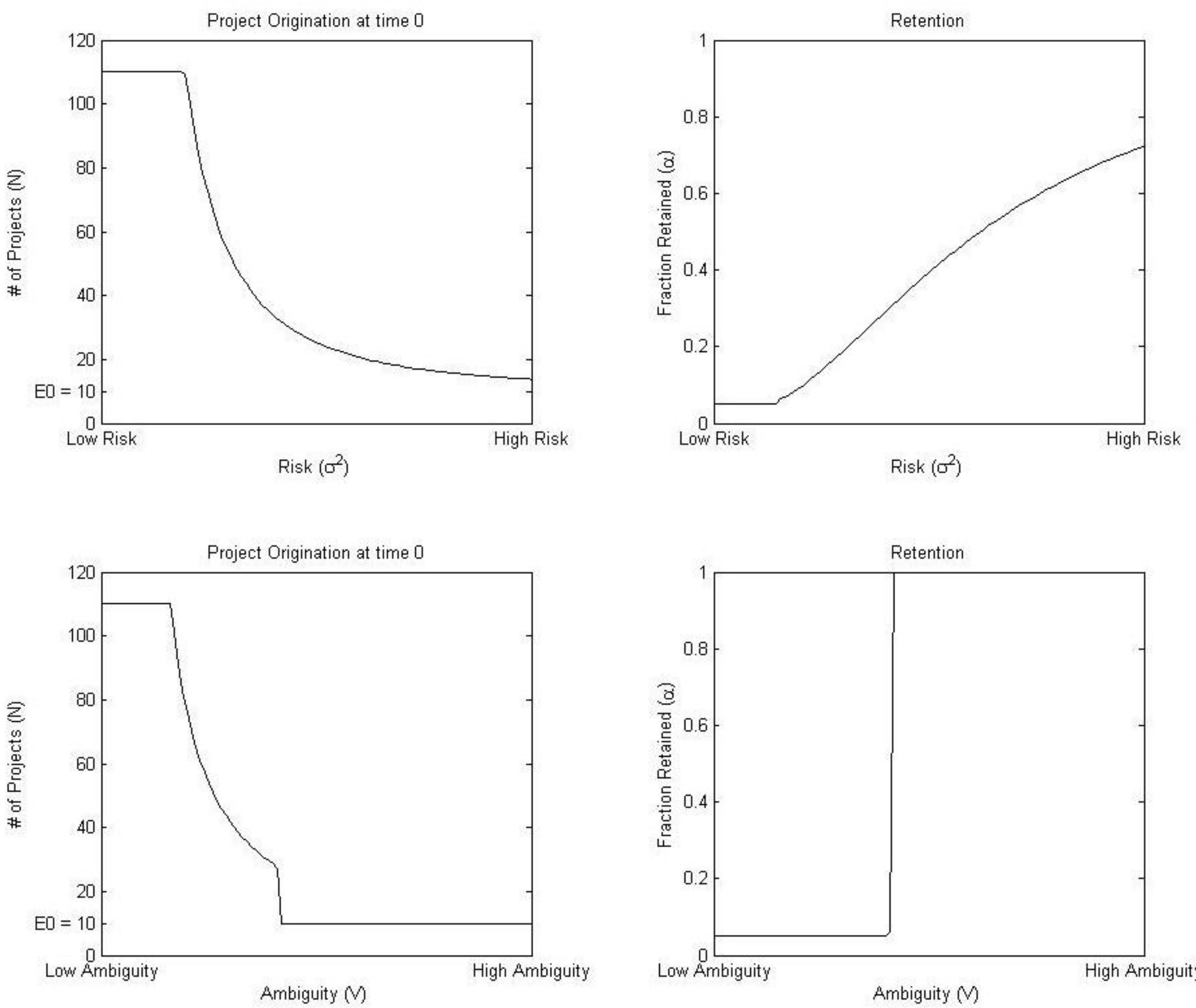
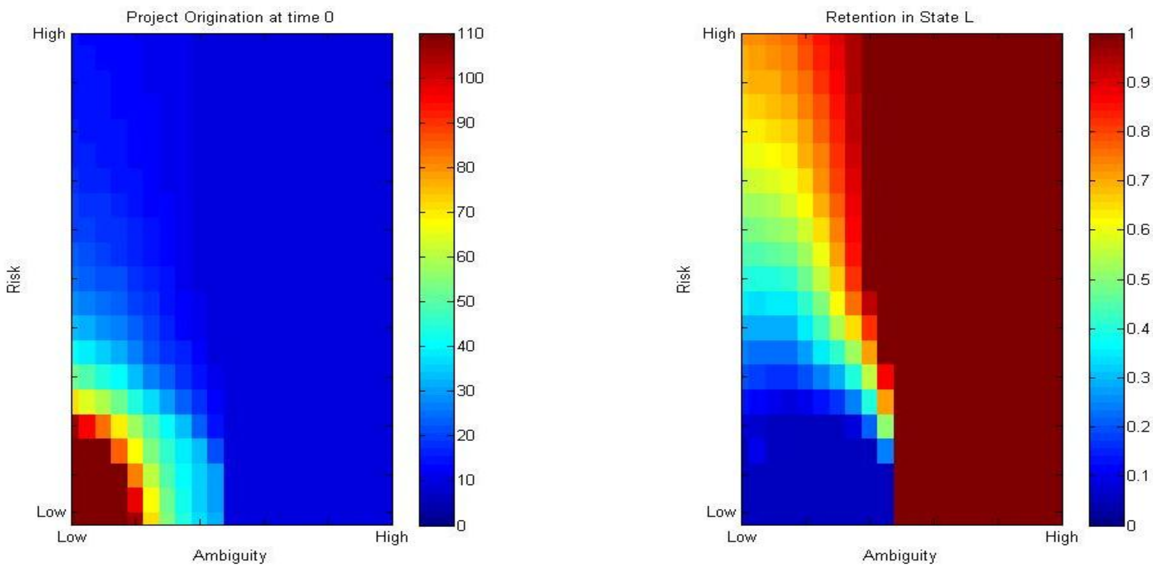


Figure 7. Ambiguity and Risk

This figure shows how the bank's optimal origination and securitization decisions are affected when there is both risk and ambiguity. Red indicates high levels of origination and retention, while blue indicates lower levels. The model parameters are fixed as follows: $E_0 = 10$; $\hat{V} = 1.2$; $i = 0.2$; $r = 0.05$; $\theta = 0.5$; $\gamma = 1$; $\underline{\alpha} = 0.05$; $\underline{V} \in [0, \hat{V}]$, and $\sigma^2 \in [0,0.2]$.



Appendix

Proof of Proposition 1. The maximization problem faced by the bank at time 0, given in equation (14), can be restated with the following Lagrangian equation

$$\begin{aligned} L = {} & (1+i)(1-\alpha)N\left(\hat{V}-(1-\alpha)N\gamma\sigma^2\right)+\alpha N\hat{V}-N-rD+\lambda_1[E_0+D-N] \\ & +\lambda_2\left[E_0+D-N+(1-\alpha)N\left(\hat{V}-(1-\alpha)N\gamma\sigma^2\right)\right. \\ & \left.-(1+r)D\right] \end{aligned} \tag{A.1}$$

where λ_1 and λ_2 are the Lagrangian multipliers corresponding to the budget constraint and the riskless debt constraint, respectively. Solving this equation yields the following Kuhn-Tucker first-order conditions:

$$\begin{aligned} & (1+i)(1-\alpha)\hat{V}-2(1+i)(1-\alpha)^2N\gamma\sigma^2+\alpha\hat{V}-1-\lambda_1 \\ & \quad +\lambda_2\left[-1+(1-\alpha)\hat{V}-2(1-\alpha)^2N\gamma\sigma^2\right]=0 \end{aligned} \tag{A.2}$$

$$-(1+i)N\hat{V}+2(1+i)(1-\alpha)N^2\gamma\sigma^2+N\hat{V}+\lambda_2\left[-N\hat{V}+2(1-\alpha)N^2\gamma\sigma^2\right]=0 \tag{A.3}$$

$$-r+\lambda_1+\lambda_2(1-(1+r))=0 \tag{A.4}$$

$$\lambda_1[E_0+D-N]=0 \tag{A.5}$$

$$\lambda_2\left[E_0+D-N+(1-\alpha)N\left(\hat{V}-(1-\alpha)N\gamma\sigma^2\right)-(1+r)D\right]=0 \tag{A.6}$$

The solution has 4 possible cases.

(i) *Neither constraint is binding*

From (A.3),

$$\alpha = 1-\frac{i\hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.7}$$

Plugging (A.7) into (A.2), given that $\lambda_1 = 0$ and $\lambda_2 = 0$ from (A.5) and (A.6), respectively, yields $\hat{V} = 1$. If $\hat{V} \geq 1$, N should be maximized, which contradicts constraint 1 not binding. If $\hat{V} < 1$, $N = 0$ at the optimum. Therefore, $D_{NA} = 0,$ and $\alpha_{NA} = 1$.

(ii) *Constraint 1 binds, constraint 2 does not*

From (A.5),

$$N = E_0 + D \tag{A.8}$$

From (A.3),

$$\alpha = 1 - \frac{i\hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.9}$$

Equation (A.4) yields the following Lagrangian multiplier

$$\lambda_1 = r \tag{A.10}$$

Plugging (A.9) and (A.10) into (A.2) yields $\hat{V} = 1 + r$. When $\hat{V} > (1 + r)$, *N* should be maximized, which contradicts constraint 2 not binding. Therefore, this case is only applicable when $\hat{V} \leq (1 + r)$. Here, any combination of *N* and $\alpha \in [\underline{\alpha}, 1]$ satisfying (A.9) will be optimal.

(iii) Constraint 2 binds, constraint 1 does not

This case is not feasible. If constraint 1 is slack, the optimal choice of *D* is 0 since there is no reason to take on costly deposits when the bank has remaining equity. Since constraint 1 is not binding, $E_0 - N > 0$. Therefore, to satisfy (A.6), we need $(1-\alpha)N\left(\hat{V} - (1-\alpha)N\gamma\sigma^2\right) < 0$. Since price is always positive, this requires $\alpha > 1$, which is not feasible given that α is a fraction bounded above by 1. Therefore, constraint 2 cannot bind before constraint 1.

(iv) Both constraints bind

From (A.5),

$$N = E_0 + D \tag{A.11}$$

From (A.6),

$$\alpha = 1 - \frac{\hat{V} - \sqrt{\hat{V}^2 - 4\gamma\sigma^2(1+r)(N-E_0)}}{2N\gamma\sigma^2} \tag{A.12}$$

Equations (A.3) and (A.4), respectively, yield the following expressions for the Lagrangian multipliers

$$\lambda_2 = \frac{2(1+i)(1-\alpha)N\gamma\sigma^2 - i\hat{V}}{\hat{V} - 2(1-\alpha)N\gamma\sigma^2} \tag{A.13}$$

$$\lambda_1 = r(1+\lambda_2) \tag{A.14}$$

Plugging (A.13) and (A.14) into (A.2) yields

$$\alpha = 1 - \frac{\hat{V}(\hat{V} + (i-1)(1+r))}{2N\gamma\sigma^2(\hat{V} + i(1+r))} \tag{A.15}$$

From (A.12), (A.15), and (A.11), the bank's optimal origination and borrowing decisions can be determined, as follows, provided $\alpha \geq \underline{\alpha}$.

$$N = E_0 + \frac{\hat{V}^2\big(\hat{V} + i(1+r)\big)^2 - \big(\hat{V}(1+r)\big)^2}{4\gamma\sigma^2(1+r)\big(\hat{V} + i(1+r)\big)^2} \tag{A.16}$$

$$D = \frac{\hat{V}^2\big(\hat{V} + i(1+r)\big)^2 - \big(\hat{V}(1+r)\big)^2}{4\gamma\sigma^2(1+r)\big(\hat{V} + i(1+r)\big)^2} \tag{A.17}$$

If α is limited by $\underline{\alpha}$, the optimal *N* is determined by setting (A.12) equal to $\underline{\alpha}$. This yields

$$N(\underline{\alpha}) = \frac{(1-\underline{\alpha})\hat{V} - (1+r) + \sqrt{\Big(-(1-\underline{\alpha})\hat{V} + (1+r)\Big)^2 - 4\gamma\sigma^2(1-\underline{\alpha})^2(1+r)E_0}}{2(1-\underline{\alpha})\gamma\sigma^2} \tag{A.18}$$

The more loans the bank originates, the more it must sell to meet its debt constraint. Therefore, when retention is at its minimum $\underline{\alpha}$, origination is at its maximum. That is, $N(\underline{\alpha})$, as defined in (A.18), is greater than *N*, as defined in (A.16) when $\alpha > \underline{\alpha}$.

When $\hat{V} > (1+r)$, the optimal solution is always case *(iv)*. This is the case I focus on since it is logical that the return on the risky loans is greater than the return on riskless debt. However, this is not necessarily the case. If $\hat{V} < 1$, the expected return to creating a new loan is less than the cost and the bank will opt not to originate any loans. The trivial solution of $N = 0, D = 0$, and $\alpha = 1$ will prevail for any model parameters. This is case *(i)*. When $1 \leq \hat{V} \leq (1+r)$, both case *(ii)* and case *(iv)* are possible.

The threshold between case *(ii)* and case *(iv)* when $1 \leq \hat{V} \leq (1+r)$ is determined by plugging (A.9) and $\underline{\alpha}$ into constraint 2. This gives the threshold

$$V^* = \frac{i(1+i)(1+r) - (1+i)\sqrt{(i(1+r))^2 - 4\gamma\sigma^2(1+i)\big(1-\underline{\alpha}\big)^2(1+r)E_0}}{i(2+1)(1-\underline{\alpha})} \tag{A.19}$$

When $\hat{V} > V^*$, loans are more valuable so the bank will originate more. This will require more deposits, which will lead the bank's debt constraint to bind. This is case *(iv)*. On the other hand, when $\hat{V} \leq V^*$, the bank originates fewer loans and the debt constraint will not bind. This is case *(ii)*.

Proof of Proposition 2. The maximization problem faced by the bank at time 0, given in equation (15), can be restated with the following Lagrangian equation

$$\begin{aligned} L = \ & \theta\big[(1+i)(1-\alpha_H)N\big(\hat{V} - (1-\alpha_H)N\gamma\sigma^2\big) + \alpha_H N\hat{V}\big] \\ & + (1-\theta)\big[\,(1+i)(1-\alpha_L)N\big(\underline{V} - (1-\alpha_L)N\gamma\sigma^2\big) + \alpha_L N\hat{V}\big] \quad - N \\ & - rD + \lambda_1[E_0 + D - N] \\ & + \lambda_2\big[E_0 + D - N + (1-\alpha_H)N\big(\hat{V} - (1-\alpha_H)N\gamma\sigma^2\big) - (1+r)D\big] \\ & + \lambda_3\big[E_0 + D - N + (1-\alpha_L)N\big(\underline{V} - (1-\alpha_L)N\gamma\sigma^2\big) \\ & - (1+r)D\big] \end{aligned} \tag{A.20}$$

where λ_1, λ_2, and λ_3 are the Lagrangian multipliers corresponding to the budget constraint, the riskless debt constraint in state *H*, and the riskless debt constrain in state *L*, respectively. Solving this equation yields the following Kuhn-Tucker first-order conditions:

$$\begin{aligned} & \theta\big[(1+i)(1-\alpha_H)\hat{V} - 2(1+i)(1-\alpha_H)^2 N\gamma\sigma^2 + \alpha_H\hat{V}\big] \\ & \quad + (1-\theta)\big[(1+i)(1-\alpha_L)\underline{V} - 2(1+i)(1-\alpha_L)^2 N\gamma\sigma^2 + \alpha_L\hat{V}\big] - 1 - \lambda_1 \\ & \quad + \lambda_2\big[-1 + (1-\alpha_H)\hat{V} - 2(1-\alpha_H)^2 N\gamma\sigma^2\big] \\ & \quad + \lambda_3\big[-1 + (1-\alpha_L)\hat{V} - 2(1-\alpha_L)^2 N\gamma\sigma^2\big] = 0 \end{aligned} \tag{A.21}$$

$$\theta\left[-(1+i)N\hat{V} + 2(1+i)(1-\alpha_H)N^2\gamma\sigma^2 + N\hat{V} + \lambda_2\big[-N\hat{V} + 2(1-\alpha_H)N^2\gamma\sigma^2\big]\right] = 0 \tag{A.22}$$

$$(1-\theta)\left[-(1+i)N\underline{V} + 2(1+i)(1-\alpha_L)N^2\gamma\sigma^2 + N\hat{V} + \lambda_3\big[-N\underline{V} + 2(1-\alpha_L)N^2\gamma\sigma^2\big]\right] = 0 \tag{A.23}$$

$$-r + \lambda_1 + r\lambda_2 + r\lambda_3 = 0 \tag{A.24}$$

$$\lambda_1[E_0 + D - N] = 0 \tag{A.25}$$

$$\lambda_2\big[E_0 + D - N + (1-\alpha_H)N\big(\hat{V} - (1-\alpha_H)N\gamma\sigma^2\big) - (1+r)D\big] = 0 \tag{A.26}$$

$$\lambda_3\big[E_0 + D - N + (1-\alpha_L)N\big(\underline{V} - (1-\alpha_L)N\gamma\sigma^2\big) - (1+r)D\big] = 0 \tag{A.27}$$

The solution has 8 possible cases, some of which are not feasible.

(i) *None of the constraints are binding*

From (A.22),

$$\alpha_H = 1 - \frac{i\hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.28}$$

From (A.23),

$$\alpha_L = 1 - \frac{(1+i)\underline{V} - \hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.29}$$

Plugging (A.28) and (A.29) into (A.21), given that $\lambda_1 = \lambda_2 = \lambda_3 = 0$ from (A.25), (A.26), and (A.27), respectively, yields $\hat{V} = 1$. If $\hat{V} \geq 1$, N should be maximized, which contradicts constraint 3 not binding. If $\hat{V} < 1$, $N = 0$ at the optimum. Therefore, $D = 0$, $\alpha_H = 1$, and $\alpha_L = 1$.

(ii) Constraint 2 binds, constraints 1 and 3 do not

This case is not feasible. If constraint 1 is slack, the optimal choice of D is 0 since there is no reason to take on costly deposits when the bank has remaining equity. Therefore, when constraint 2 is binding, we have

$$E_0 - N + (1-\alpha_H)N\left(\hat{V} - (1-\alpha_H)N\gamma\sigma^2\right) = 0 \tag{A.30}$$

Since constraint 1 is not binding, $E_0 - N > 0$. Therefore, to satisfy (A.26), we need $(1-\alpha_H)N\left(\hat{V} - (1-\alpha_H)N\gamma\sigma^2\right) < 0$. Since price is always positive, this requires $\alpha_H > 1$, which is not feasible given that α_H is a fraction bounded above by 1. Therefore, constraint 2 cannot bind before constraint 1.

(iii) Constraint 3 binds, constraints 1 and 2 do not

By the same logic as shown in case (*ii*), this case is not feasible.

(iv) Constraint 1 binds, constraints 2 and 3 do not

From (A.25),

$$N = E_0 + D \tag{A.31}$$

From (A.22),

$$\alpha_H = 1 - \frac{i\hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.32}$$

From (A.23),

$$\alpha_L = 1 - \frac{(1+i)\underline{V} - \hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.33}$$

Equation (A.24) yields the following Lagrangian multiplier

$$\lambda_1 = r \tag{A.34}$$

Plugging (A.32) and (A.33) into (A.21) yields $\hat{V} = 1 + r$. When $\hat{V} > (1 + r)$, N should be maximized, which contradicts constraints 2 or 3 not binding. Therefore, this case is only applicable when $\hat{V} \leq (1 + r)$. Here, any combination of N and $(\alpha_H, \alpha_L) \in [\underline{\alpha}, 1]$ satisfying (A.32) and (A.33) will be optimal.

(v) Constraints 2 and 3 bind, constraint 1 does not

By the same logic as shown in case (*ii*), this case is not feasible.

(vi) Constraints 1 and 2 bind, constraint 3 does not

From (A.25),

$$N = E_0 + D \tag{A.35}$$

From (A.26),

$$\alpha_H = 1 - \frac{\hat{V} - \sqrt{\hat{V}^2 - 4\gamma\sigma^2(N - E_0 + rD)}}{2N\gamma\sigma^2} \tag{A.36}$$

From (A.23),

$$\alpha_L = 1 - \frac{(1+i)\underline{V} - \hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.37}$$

Equations (A.22) and (A.24), respectively, yield the following expressions for the Lagrangian multipliers

$$\lambda_2 = \frac{\theta\left[2(1+i)(1-\alpha_L)N\gamma\sigma^2 - i\hat{V}\right]}{\hat{V} - 2(1-\alpha_L)N\gamma\sigma^2} \tag{A.38}$$

$$\lambda_1 = r(1+\lambda_2) \tag{A.39}$$

Plugging (A.37), (A.38), and (A.39) into (A.21) yields

$$\alpha_H = 1 - \frac{\hat{V}(\hat{V} + (1+r)(1-(1+i)\theta)) - \theta(1+r)\hat{V}}{2N\gamma\sigma^2(\hat{V} + (1+r)(1-(1+i)\theta))} \tag{A.40}$$

From (A.35), (A.36), and (A.40), the bank's optimal origination and borrowing decisions can be determined, as follows, provided $\alpha \geq \underline{\alpha}$.

$$N = E_0 + \frac{\hat{V}^2(\hat{V} + (1+r)(1-(1+i)\theta))^2 - (\theta(1+r)\hat{V})^2}{4\gamma\sigma^2(1+r)(\hat{V} + (1+r)(1-(1+i)\theta))^2} \tag{A.41}$$

Given that constraint 1 and 2 bind, but constraint 3 does not, this requires

$$(1+r)D = (1-\alpha_H)N\big(\hat{V} - (1-\alpha_H)N\gamma\sigma^2\big) < (1-\alpha_L)N\big(\hat{V} - (1-\alpha_L)N\gamma\sigma^2\big) \tag{A.42}$$

Plugging (A.36), (A.37), and (A.41) into (A.42) yields

$$\hat{V}^2 - \frac{\theta^2(1+r)^2\hat{V}^2}{\big(\hat{V} + (1+r)(1-(1+i)\theta)\big)^2} < \underline{V}^2 - \frac{\hat{V}^2}{(1+i)^2} \tag{A.43}$$

This inequality does not hold for any parameter values, meaning case *(vi)* is not feasible. The debt constraint in state *L* always binds before the debt constraint in state *H*.

(vii) Constraints 1 and 3 bind, constraint 2 does not

From (A.25),

$$N = E_0 + D \tag{A.44}$$

From (A.22),

$$\alpha_H = 1 - \frac{i\hat{V}}{2(1+i)N\gamma\sigma^2} \tag{A.45}$$

From (A.27),

$$\alpha_L = 1 - \frac{\underline{V} - \sqrt{\underline{V}^2 - 4\gamma\sigma^2(N - E_0 + rD)}}{2N\gamma\sigma^2} \tag{A.46}$$

Equations (A.23) and (A.24), respectively, yield the following expressions for the Lagrangian multipliers

$$\lambda_3 = \frac{(1-\theta)\left[2(1+i)(1-\alpha_L)N\gamma\sigma^2 + \hat{V} - (1+i)\underline{V}\right]}{\underline{V} - 2(1-\alpha_L)N\gamma\sigma^2} \tag{A.47}$$

$$\lambda_1 = r(1+\lambda_3) \tag{A.48}$$

Plugging (A.45), (A.47), and (A.48) into (A.21) yields

$$\alpha_L = 1 - \frac{\underline{V}(\hat{V} + (1+r)(1-(1+i)(1-\theta))) - (1-\theta)(1+r)\hat{V}}{2N\gamma\sigma^2(\hat{V} + (1+r)(1-(1+i)(1-\theta)))} \tag{A.49}$$

From (A.44), (A.46), and (A.49), the bank's optimal origination and borrowing decisions can be determined, as follows, provided $\alpha \geq \underline{\alpha}$.

$$N = E_0 + \frac{\underline{V}^2(\hat{V} + (1+r)(1-(1+i)(1-\theta)))^2 - ((1-\theta)(1+r)\hat{V})^2}{4\gamma\sigma^2(1+r)(\hat{V} + (1+r)(1-(1+i)(1-\theta)))^2} \tag{A.50}$$

$$D = \frac{\underline{V}^2(\hat{V} + (1+r)(1-(1+i)(1-\theta)))^2 - ((1-\theta)(1+r)\hat{V})^2}{4\gamma\sigma^2(1+r)(\hat{V} + (1+r)(1-(1+i)(1-\theta)))^2} \tag{A.51}$$

(viii) All constraints bind

From (A.25),

$$N = E_0 + D \tag{A.52}$$

From (A.26),

$$\alpha_H = 1 - \frac{\hat{V} - \sqrt{\hat{V}^2 - 4\gamma\sigma^2(N - E_0 + rD)}}{2N\gamma\sigma^2} \tag{A.53}$$

From (A.27),

$$\alpha_L = 1 - \frac{\underline{V} - \sqrt{\underline{V}^2 - 4\gamma\sigma^2(N - E_0 + rD)}}{2N\gamma\sigma^2} \tag{A.54}$$

Equations (A.22), (A.23), and (A.24), respectively, yield the following expressions for the Lagrangian multipliers

$$\lambda_2 = \frac{\theta\left[2(1+i)(1-\alpha_H)N\gamma\sigma^2 - i\hat{V}\right]}{\hat{V} - 2(1-\alpha_H)N\gamma\sigma^2} \tag{A.55}$$

$$\lambda_3 = \frac{(1-\theta)\left[2(1+i)(1-\alpha_L)N\gamma\sigma^2 + \hat{V} - (1+i)\underline{V}\right]}{\underline{V} - 2(1-\alpha_L)N\gamma\sigma^2} \tag{A.56}$$

$$\lambda_1 = r(1+\lambda_2+\lambda_3) \tag{A.57}$$

Plugging (A.52), (A.54), (A.55), (A.56), and (A.57) into (A.21) yields

$$\alpha_H = 1 - \frac{\hat{V}}{2N\gamma\sigma^2} - \frac{\theta(1+r)\hat{V}}{2N\gamma\sigma^2\left[\hat{V} + i(1+r) - \frac{(1-\theta)(1+r)\hat{V}}{\sqrt{\underline{V}^2 - 4\gamma\sigma^2(1+r)(N-E_0)}}\right]} \quad \text{(A.58)}$$

Setting (A.53) and (A.58) equal yields the bank's optimal origination decision.

When $\hat{V} > (1+r)$, the bank's optimal solution falls in either case *(vii)* or case *(viii)*. Plugging α_H from case *(vii)*, as defined by (A.45), into constraint 2 shows that, for the constraint to be slack, the following condition is required.

$$N < E_0 + \frac{2i\hat{V}^2 + i^2\hat{V}^2}{4(1+i)^2(1+r)\gamma\sigma^2} \quad \text{(A.59)}$$

Setting (A.59) equal to (A.50) yields the following threshold on $\underline{V}$.

$$\underline{V}^* = \hat{V}\sqrt{\frac{((1-\theta)(1+r)(1+i))^2 + (2i+i^2)\left(\hat{V} + i(1+r)\right)^2}{(1+i)^2\left(\hat{V} + i(1+r)\right)^2}} \quad \text{(A.60)}$$

Therefore, when $\underline{V} < \underline{V}^*$, ambiguity is high, origination is low, and constraint 2 does not bind. However, when $\underline{V} \geq \underline{V}^*$, constraint 2 will bind as the bank is taking on more deposits to fund more loans in this lower ambiguity setting.

Proof of Proposition 3. When there is no risk in the market, the bank's maximization problem at time 0, given in equation (17), can be restated with the following Lagrangian equation

$$\begin{aligned} L = {} & \theta\left[(1+i)(1-\alpha_H)N\hat{V} + \alpha_H N\hat{V}\right] + (1-\theta)\left[(1+i)(1-\alpha_L)N\underline{V} + \alpha_L N\hat{V}\right] - N \\ & - rD + \lambda_1[E_0 + D - N] + \lambda_2\left[E_0 + D - N + (1-\alpha_H)N\hat{V} - (1+r)D\right] \\ & + \lambda_3\left[E_0 + D - N + (1-\alpha_L)N\underline{V} - (1+r)D\right] \end{aligned} \quad \text{(A.61)}$$

where λ_1, λ_2, and λ_3 are the Lagrangian multipliers corresponding to the budget constraint, the riskless debt constraint in state *H*, and the riskless debt constrain in state *L*, respectively. Solving this yields the following Kuhn-Tucker first-order conditions:

$$\begin{aligned} & \theta\left[(1+i)(1-\alpha_H)\hat{V} + \alpha_H\hat{V}\right] + (1-\theta)\left[(1+i)(1-\alpha_L)\underline{V} + \alpha_L\hat{V}\right] - 1 - \lambda_1 \\ & + \lambda_2\left[-1 + (1-\alpha_H)\hat{V}\right] + \lambda_3\left[-1 + (1-\alpha_L)\hat{V}\right] = 0 \end{aligned} \quad \text{(A.62)}$$

$$\theta\left[-(1+i)N\hat{V} + N\hat{V} + \lambda_2\left[-N\hat{V}\right]\right] = 0 \quad \text{(A.63)}$$

$$(1-\theta)\left[-(1+i)N\underline{V}+N\hat{V}+\lambda_3\left[-N\underline{V}\right]\right]=0 \qquad \text{(A.64)}$$

$$-r+\lambda_1+r\lambda_2+r\lambda_3=0 \qquad \text{(A.65)}$$

$$\lambda_1[E_0+D-N]=0 \qquad \text{(A.66)}$$

$$\lambda_2\left[E_0+D-N+(1-\alpha_H)N\hat{V}-(1+r)D\right]=0 \qquad \text{(A.67)}$$

$$\lambda_3\left[E_0+D-N+(1-\alpha_L)N\underline{V}-(1+r)D\right]=0 \qquad \text{(A.68)}$$

First note that, in state H, the bank will always choose $\alpha_H=\underline{\alpha}$. When there is no risk, the price the bank receives does not decrease with quantity. Therefore, the bank can receive the full value $\hat{V}$ for all securities it sells and then investment the proceeds in additional loans for an even higher return.

Also note, that as shown in the case with both risk and ambiguity, the budget constraint always binds when $\hat{V}>1$. Therefore, $N=E_0+D$.

Consider the case when constraint 3 binds, in addition to constraint 1. From (A.68), we have

$$\alpha_L=1-\frac{(1+r)(N-E_0)}{N\underline{V}} \qquad \text{(A.69)}$$

Equations (A.64) and (A.65), respectively, yield the following expressions for the Lagrangian multipliers

$$\lambda_3=\frac{(1-\theta)(\hat{V}-(1+i)\underline{V})}{\underline{V}} \qquad \text{(A.70)}$$

$$\lambda_1=r(1+\lambda_3) \qquad \text{(A.71)}$$

Plugging (A.69), (A.70), and (A.71) into (A.62) yields

$$1+r-\theta\left(1+i-\underline{\alpha}i\right)\hat{V}+(1-\theta)\left(-\hat{V}+\frac{\hat{V}-(1+i)\underline{V}}{\underline{V}}(1+r)\right)=0 \qquad \text{(A.72)}$$

Solving (A.72) for $\underline{V}$ gives us the threshold

$$V^*=\frac{(1-\theta)(1+r)\hat{V}}{\left(i\theta(1-\underline{\alpha})+1\right)\hat{V}+(1+r)(i-\theta-i\theta)} \qquad \text{(A.73)}$$

When $\underline{V}<V^*$, there is severe ambiguity in the market such that the bank will opt to retain everything in state L. That is, $\alpha_L=1$. However, if $\underline{V}\geq V^*$, the lower price received due to ambiguity is overweighed by the benefit of securitizing and investing in additional loans. Therefore, $\alpha_L=\underline{\alpha}$.

The alternative case is that constraint 3 does not bind. In this case, from (A.58), we obtain

$$\underline{V} = \frac{\hat{V}}{(1+i)} \tag{A.74}$$

That is, if $\underline{V} < \frac{\hat{V}}{(1+i)}$, the bank will choose $\alpha_L = 1$ and if $\underline{V} \geq \frac{\hat{V}}{(1+i)}$, the bank will choose $\alpha_L = \underline{\alpha}$. Therefore, to determine if constraint 3 binds or not, we need to compare $\frac{\hat{V}}{(1+i)}$ and V^*. This yields the required condition for constraint 3 to bind:

$$\hat{V}\left(1 + \theta i\left(1 - \underline{\alpha}\right)\right) > 1 + r \tag{A.75}$$

Intuitively, this condition will always hold under reasonable parameter values given that $\theta i\left(1 - \underline{\alpha}\right) > 0$ and the return on the risky asset, $\hat{V}$, should be greater than the return on riskless deposits, $1 + r$.

Given that constraint 3 always binds, the optimal value for origination is solved by plugging α_L into (A.69) and solving for N. Likewise, D is obtained from $N = E_0 + D$. Specifically, we obtain

$$N = \begin{cases} E_0 & \text{if } \underline{V} < V^* \\ \dfrac{(1+r)E_0}{1 + r - (1 - \alpha_L)\underline{V}} & \text{if } \underline{V} < V^* \end{cases} \tag{A.76}$$